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Robust Design of Axial Compressor Blades Based on Sigma-Point Method

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Motivation and Approach

- general problem: $f_{\mathbf{X}}(\mathbf{x}) \xrightarrow{\mathbf{y}=\mathbf{y}(\mathbf{x})} f_{\mathbf{Y}}(\mathbf{y})$

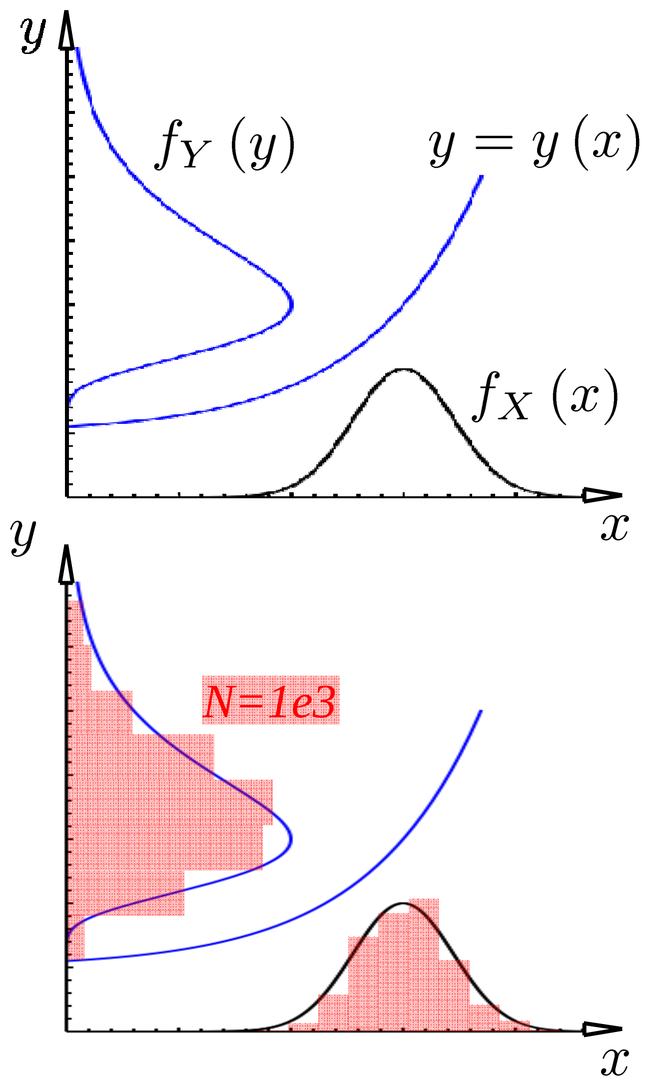
  $\mathbf{x} \in \mathbb{R}^n$ — *uncertain input* — *nonlinear mapping* — *uncertain output*

assumption: $\mathbf{y} \in \mathbb{R}$

- engineers main interests related to $f_Y(y)$:
  – expectation, e.g. $\mu_Y = \mathrm{E}[Y]$
  – scatter, e.g. $\sigma_X^2 = \mathrm{E}\left[(X-\mu_X)^2\right]$
  – probability, e.g. $P^f = P[h(\mathbf{X}) < 0]$

How to evaluate?

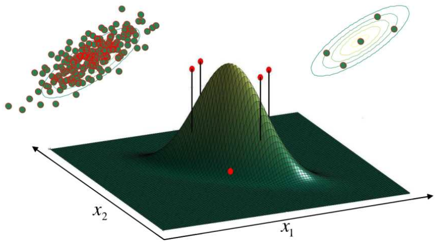
- robust approach: Monte Carlo Simulation
  (based on LHC, oLHC, DS)

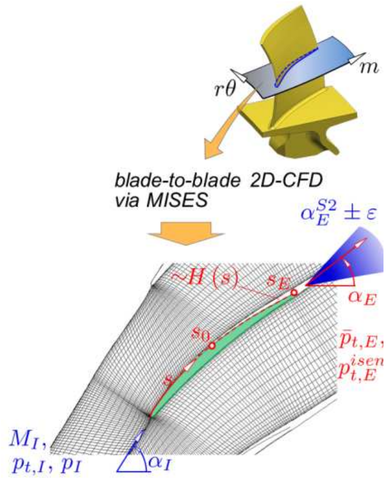


Motivation and Approach

Related to RDO: Is it possible to either reduce computational effort for given level of accuracy or to increase prediction quality with same effort?

Sigma-Point Method!

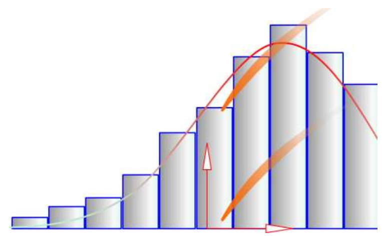




Outline

- Introduction to SPM
- Performance assessment
- How to use SPM for RDO?
- Robust aerodynamic compressor blade design
- General industrial applicability

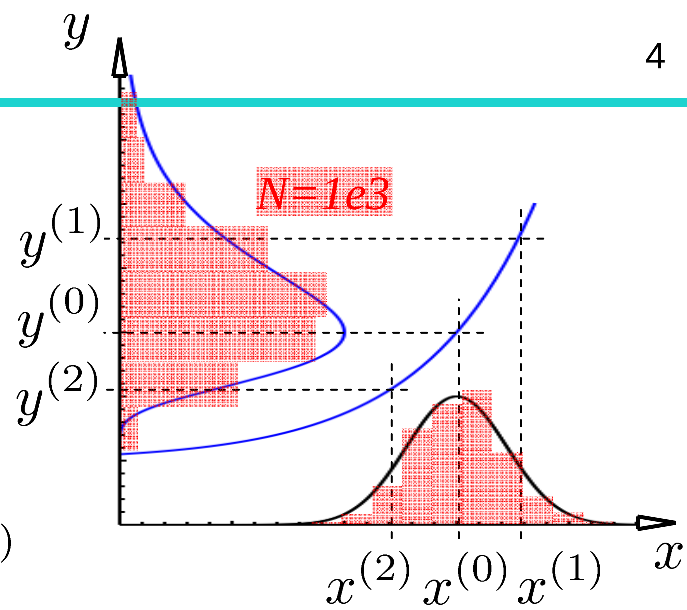
$$\min_{\mathbf{p}\in\mathcal{P}} \begin{bmatrix}\mu_Y(\mathbf{p})\\ \sigma_Y(\mathbf{p})\end{bmatrix}$$



Introduction to SP Method

- based on Gaussian Quadrature

  "…With a fixed number of parameters [Sigma Points], *it should be easier to approximate a Gaussian distribution than it is to approximate an arbitrary nonlinear function…"**



*[Julier and Uhlmann, 1996/2004]

  – use $(2n+1)$ *Sigma-Points*:

  $$\mathbf{x}^{(0)} = \mathbf{E}\left[f_{\mathbf{X}}(\mathbf{x})\right] \quad \mathbf{x}^{(i)} = \mathbf{x}^{(0)} \pm \xi \left(\sqrt{\boldsymbol{\Sigma}_{\mathbf{x}}}\right)^{(i)}$$

  – direct propagation: $\mathbf{y}^{(i)} = \mathbf{y}\left(\mathbf{x}^{(i)}\right)$

  – approximate expectation and covariance:

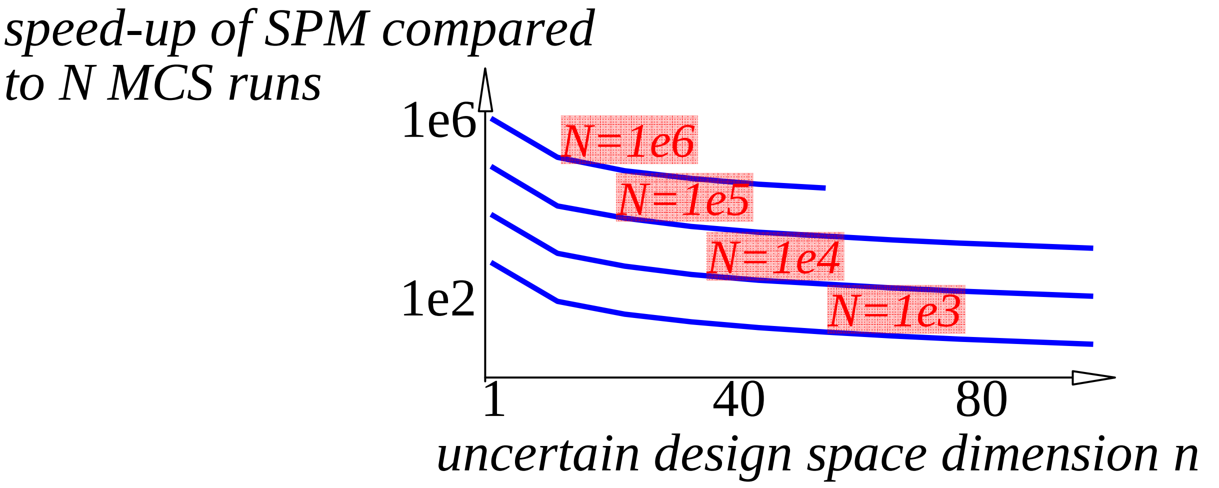
$$\mathbf{E}\left[f_{\mathbf{Y}}(\mathbf{y})\right] \approx \sum_{i=0}^{2n} w^{(i)} \mathbf{y}^{(i)}$$

$$\boldsymbol{\Sigma}_{\mathbf{y}}\left[f_{\mathbf{Y}}(\mathbf{y})\right] \approx \sum_{i=0}^{2n} w^{(i)} \left(\mathbf{y}^{(i)} - \mathbf{E}\left[f_{\mathbf{Y}}(\mathbf{y})\right]\right)\left(\mathbf{y}^{(i)} - \mathbf{E}\left[f_{\mathbf{Y}}(\mathbf{y})\right]\right)^{\mathrm{T}}$$

Introduction to SP Method

- features
  - direct propagation of uncertainties through a nonlinear/ non-monotonic system
  - deterministic, gradient free, simple implementation
  - accounts for curvature

What about speed-up?



What about prediction accuracy?

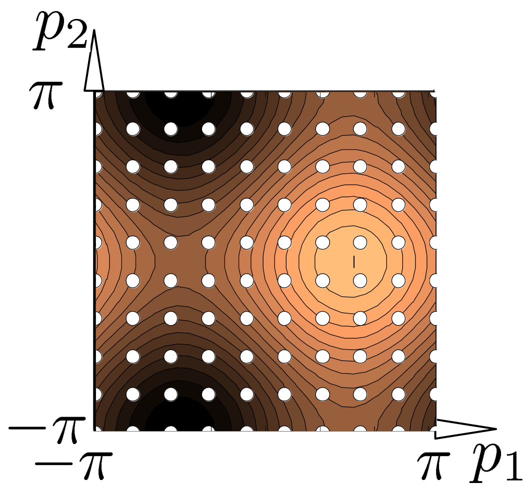
Performance assessment

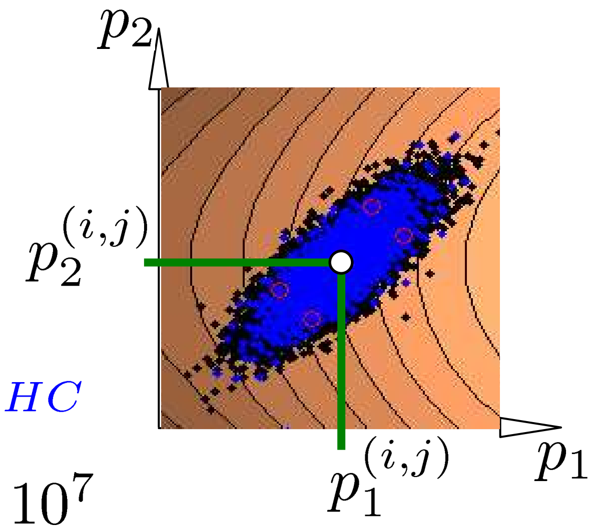
- procedure to compare *SPM* with *MCS*
  - define test problem and input variability, e.g.
    $f(\mathbf{p}) = \sin(p_1) + \cos(p_2)$ where $\mathbf{p} \in \mathbb{R}^2$,
    $-\pi \leq \mathbf{p} \leq \pi$ and $\Delta\mathbf{p} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$
  - generate full factorial design of experiment with e.g. $N_{DoE} = N_{fac}^2 = 100$
  - evaluate mean values and variances at each experiment, i.e.:

    $\mu_{SP}^{(i,j)}, \sigma_{SP}^{(i,j)}$ *estimates by SPM with* $N_{SP} = 5$

    $\mu_{MCS}^{(i,j)}, \sigma_{MCS}^{(i,j)}$ *estimates by MCS with* $N_{LHC/oLHC}$

    $\mu^{(i,j)}, \sigma^{(i,j)}$ *"exact" values by LHC with* $N = 10^7$
  - absolute compute errors: $\varepsilon_{SP}^{\mu^{(i,j)}}, \varepsilon_{SP}^{\sigma^{(i,j)}}, \varepsilon_{MCS}^{\mu^{(i,j)}}, \varepsilon_{MCS}^{\sigma^{(i,j)}}$

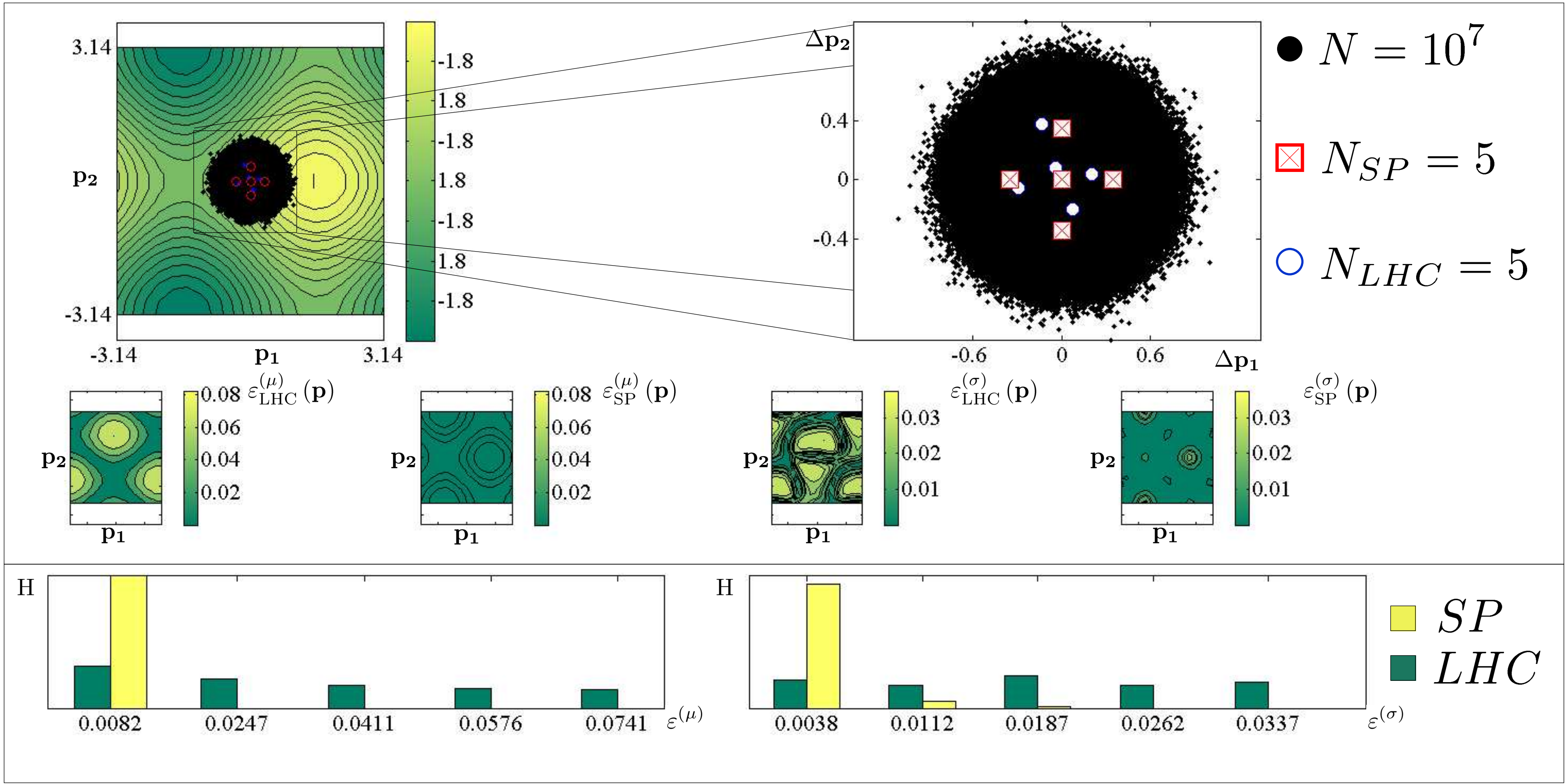




Performance assessment – SPM vs. LHC

$$f(\mathbf{p}) = \sin(p_1) + \cos(p_2) \quad \text{where } \mathbf{p} \in \mathbb{R}^2 \quad \text{and } -\pi \le \mathbf{p} \le \pi$$

Variability $\Delta\mathbf{p} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$



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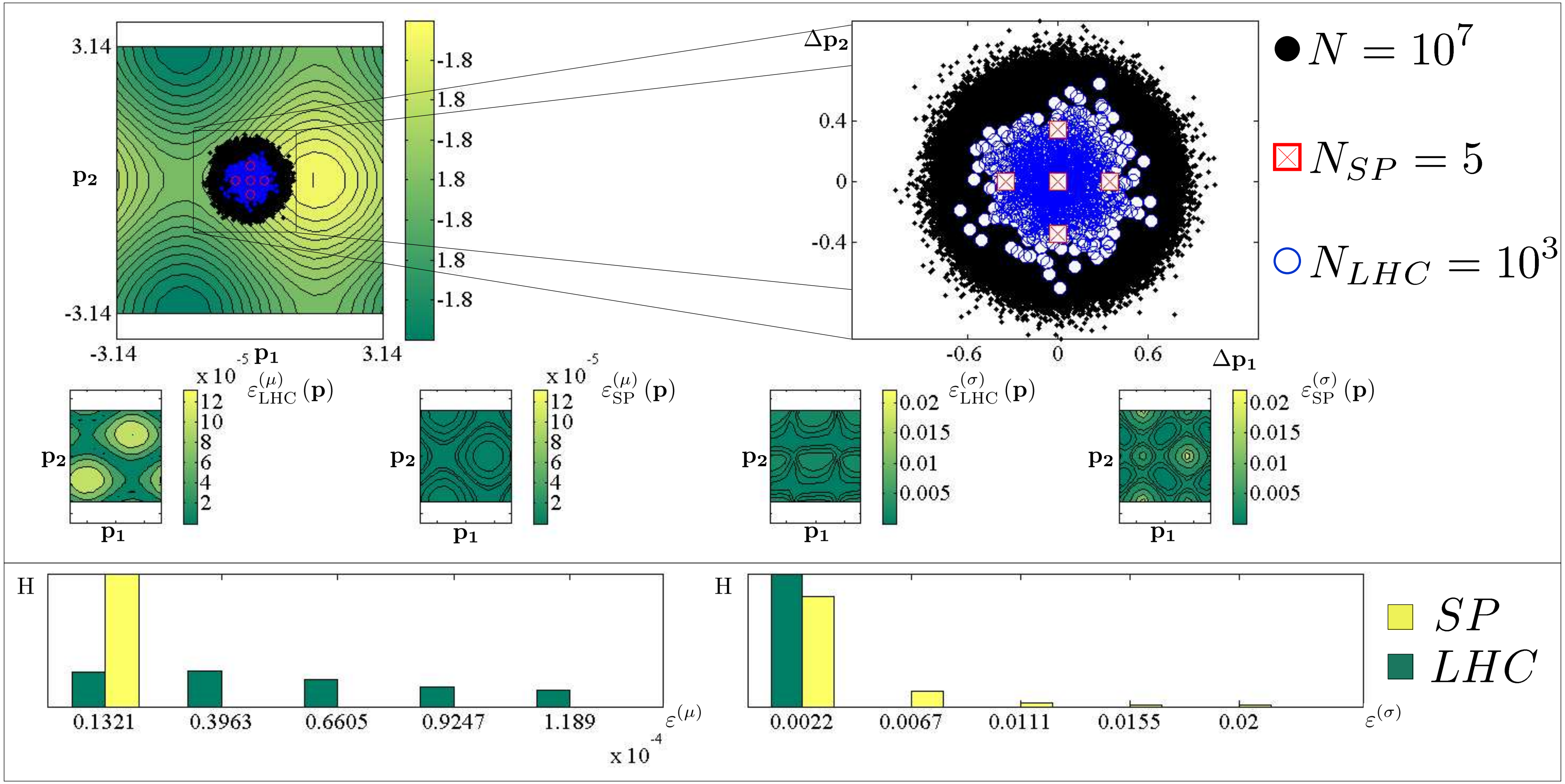

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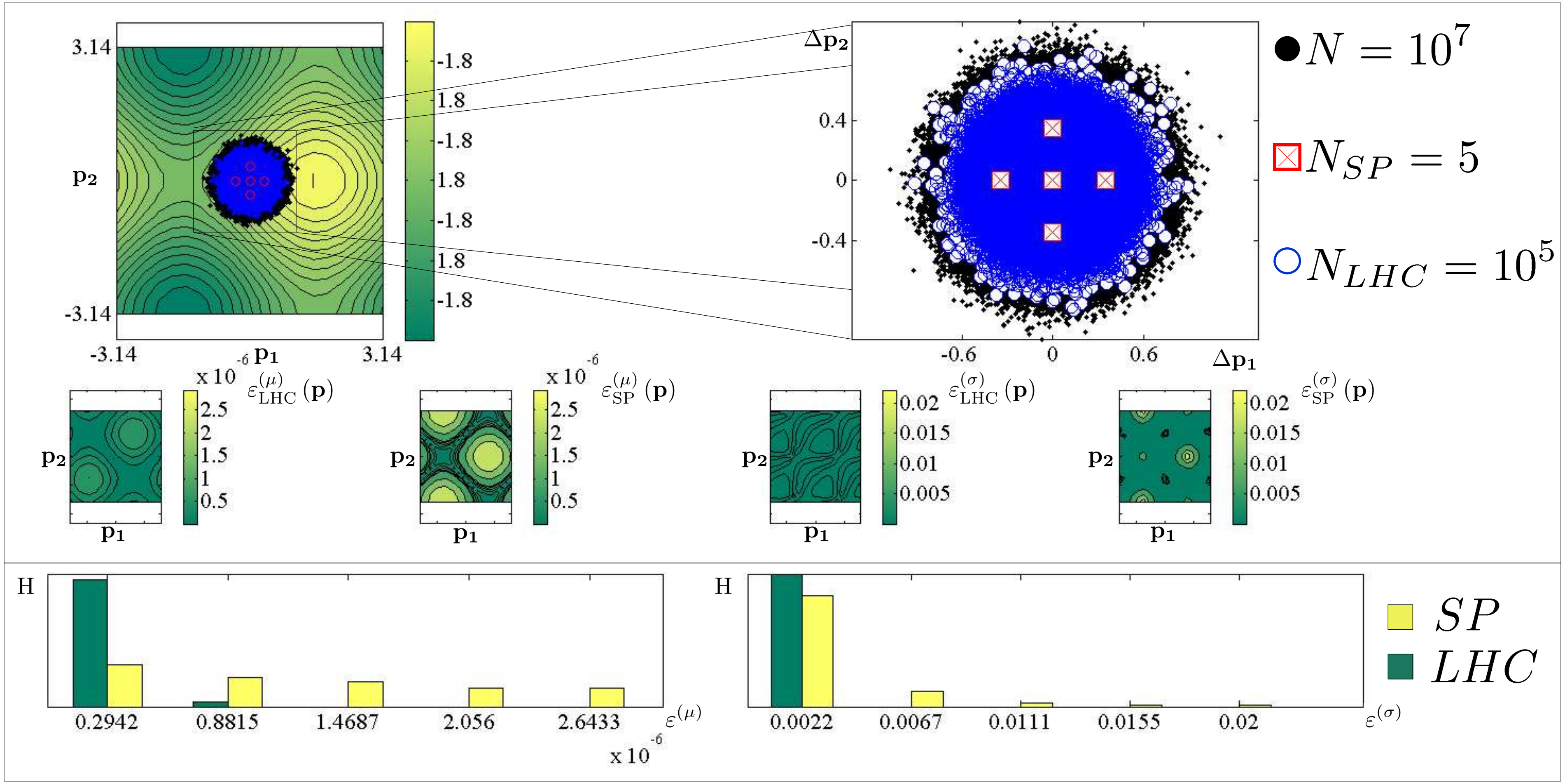
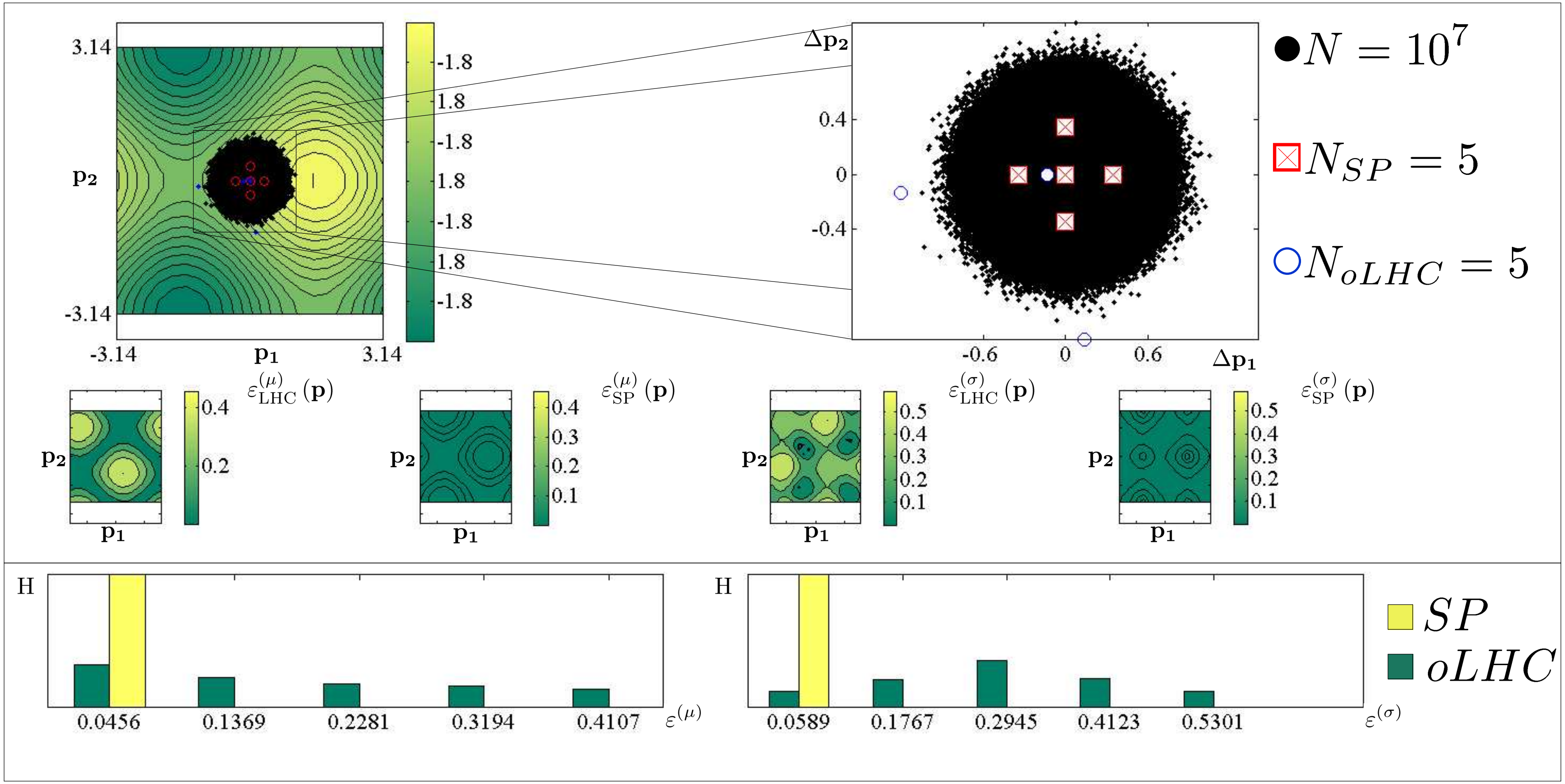


Performance assessment – SPM vs. oLHC

$$f(\mathbf{p}) = \sin(p_1) + \cos(p_2) \quad \text{where } \mathbf{p} \in \mathbb{R}^2 \quad \text{and } -\pi \leq \mathbf{p} \leq \pi$$

$$\text{Variability } \Delta\mathbf{p} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$$



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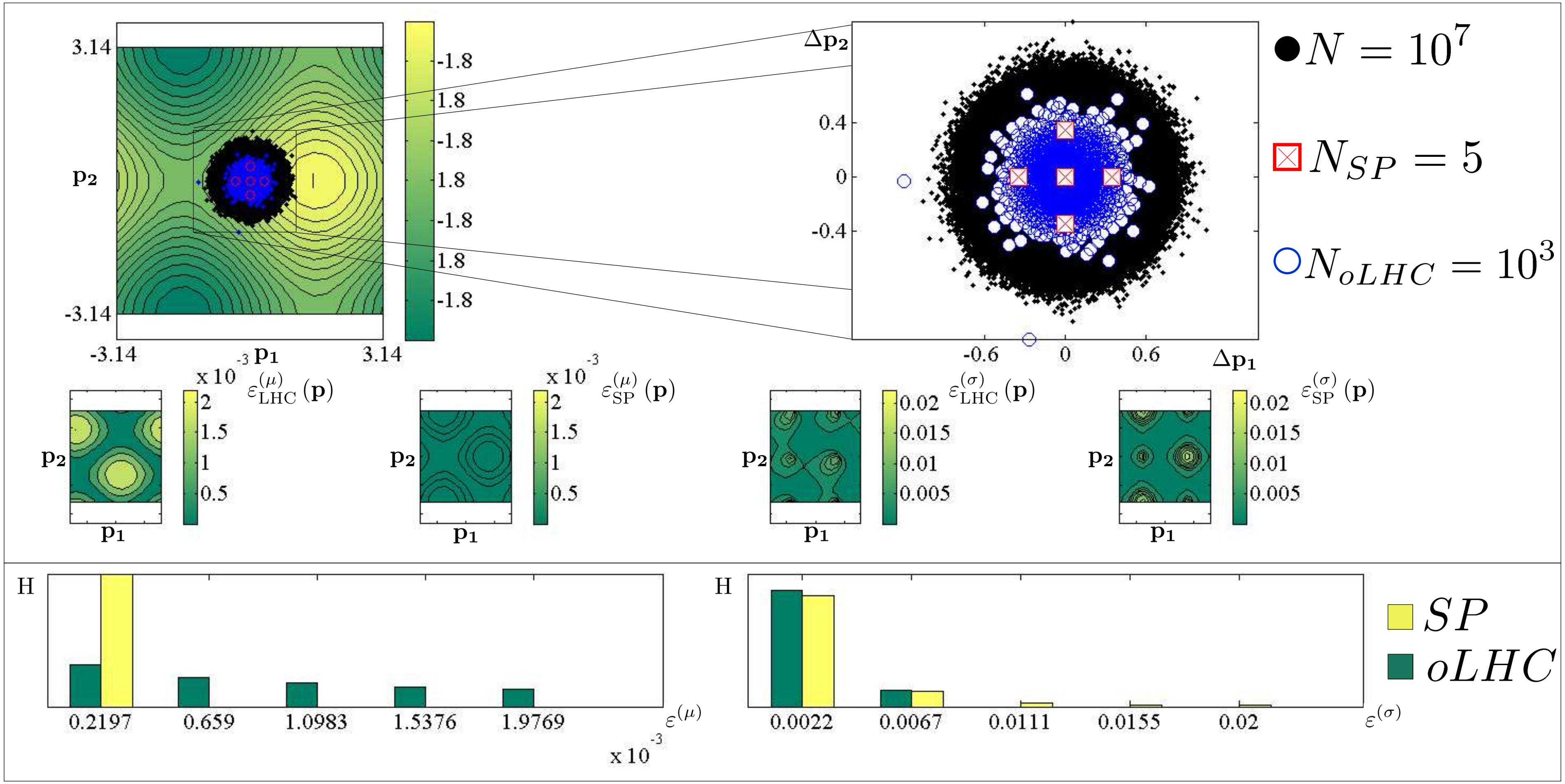
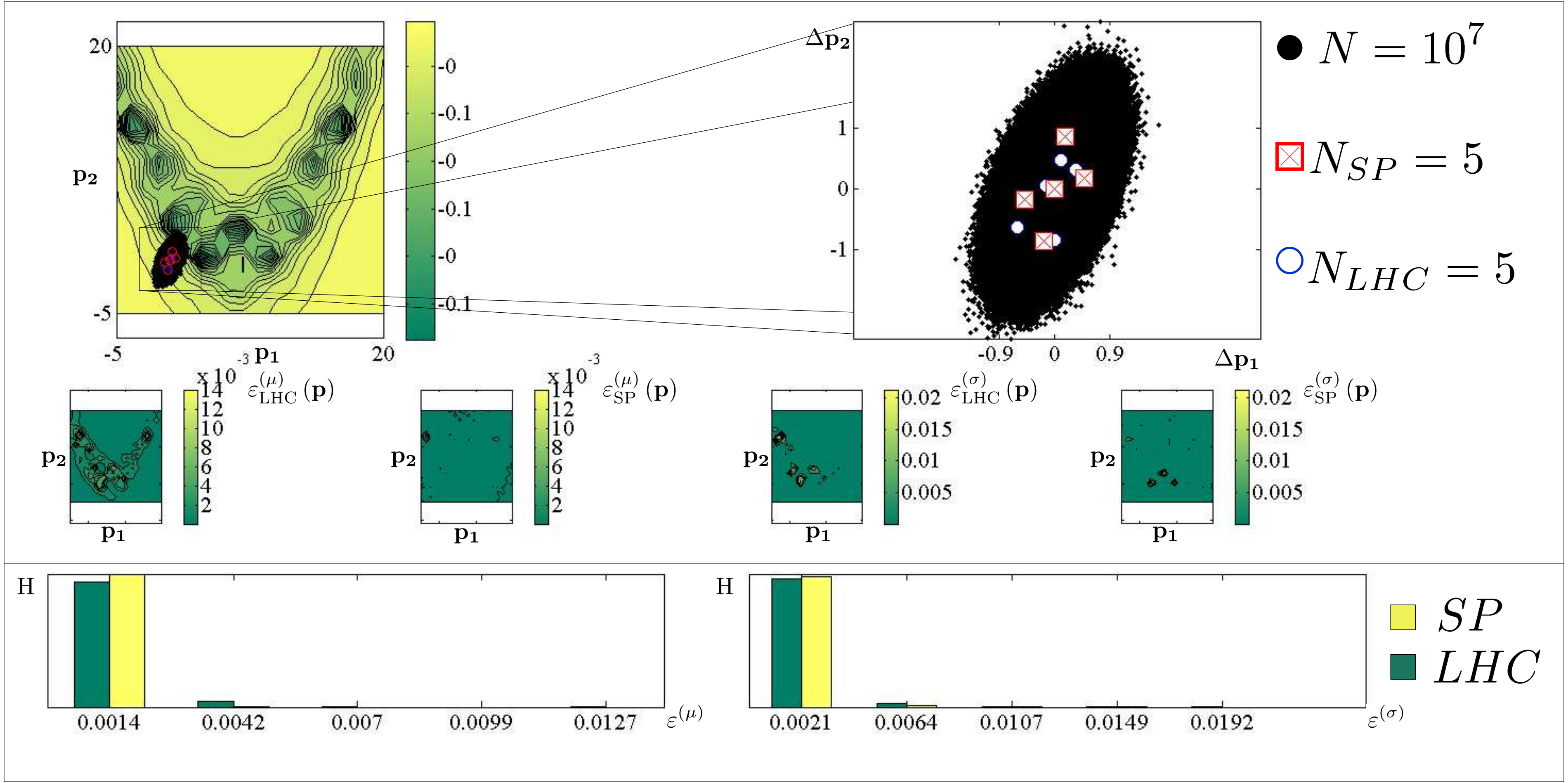


Performance assessment – SPM vs. LHC

Problem : RCOS test function

Variability $\Delta \mathbf{p} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$



Performance assessment – SPM vs. LHC

Problem : RCOS test function

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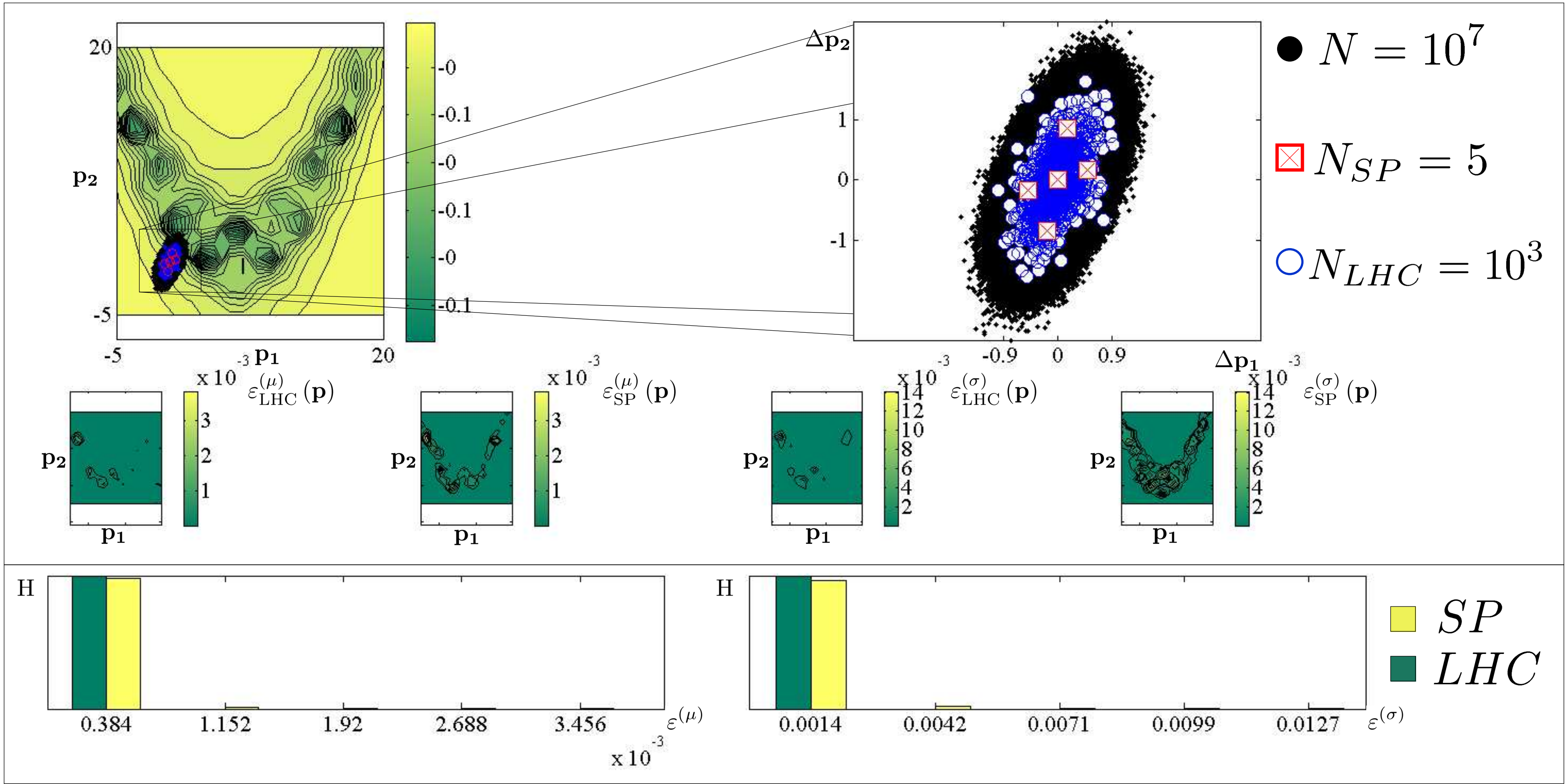
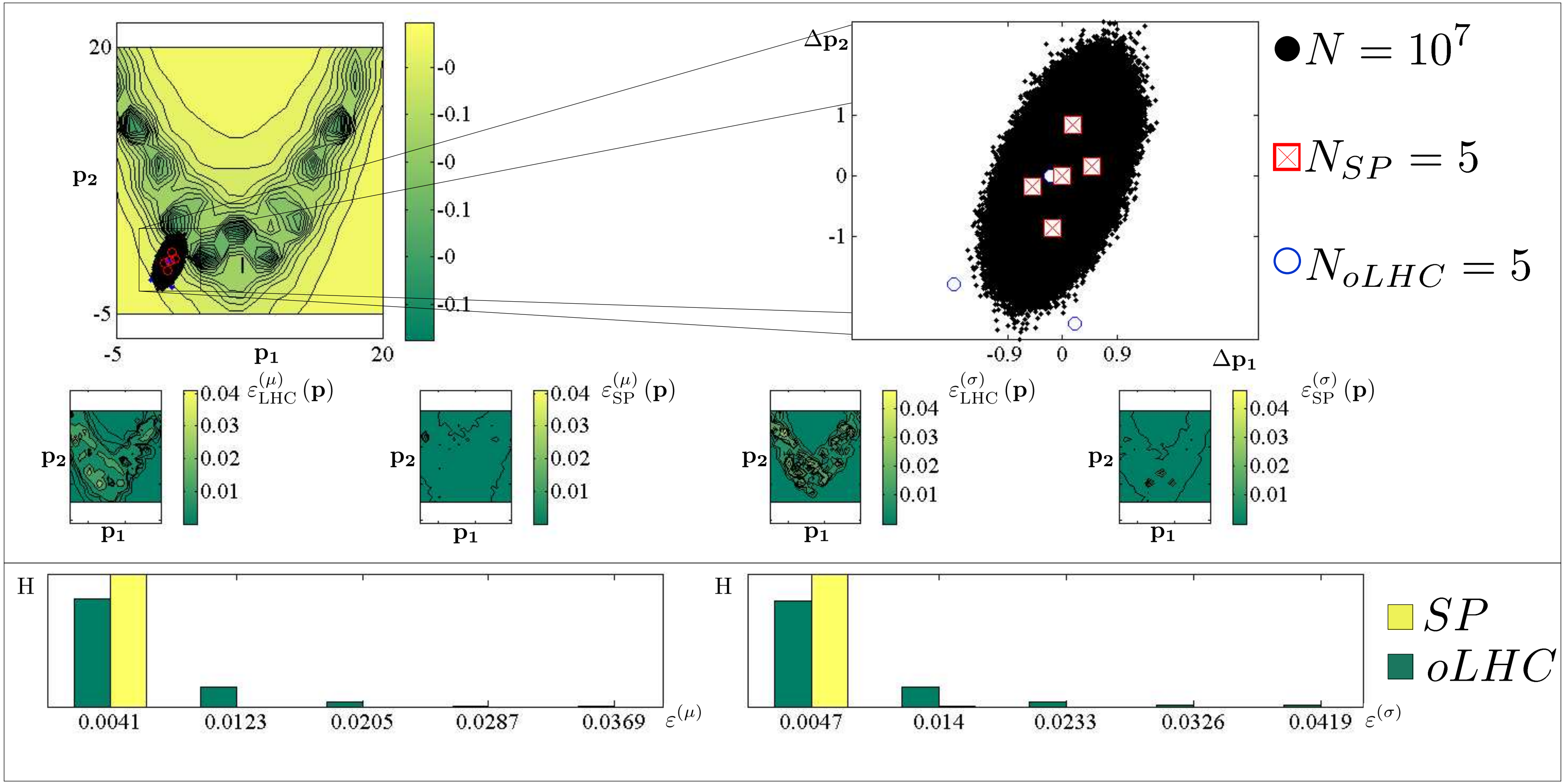


Performance assessment – SPM vs. oLHC

Problem : RCOS test function

Variability $\Delta\mathbf{p} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$

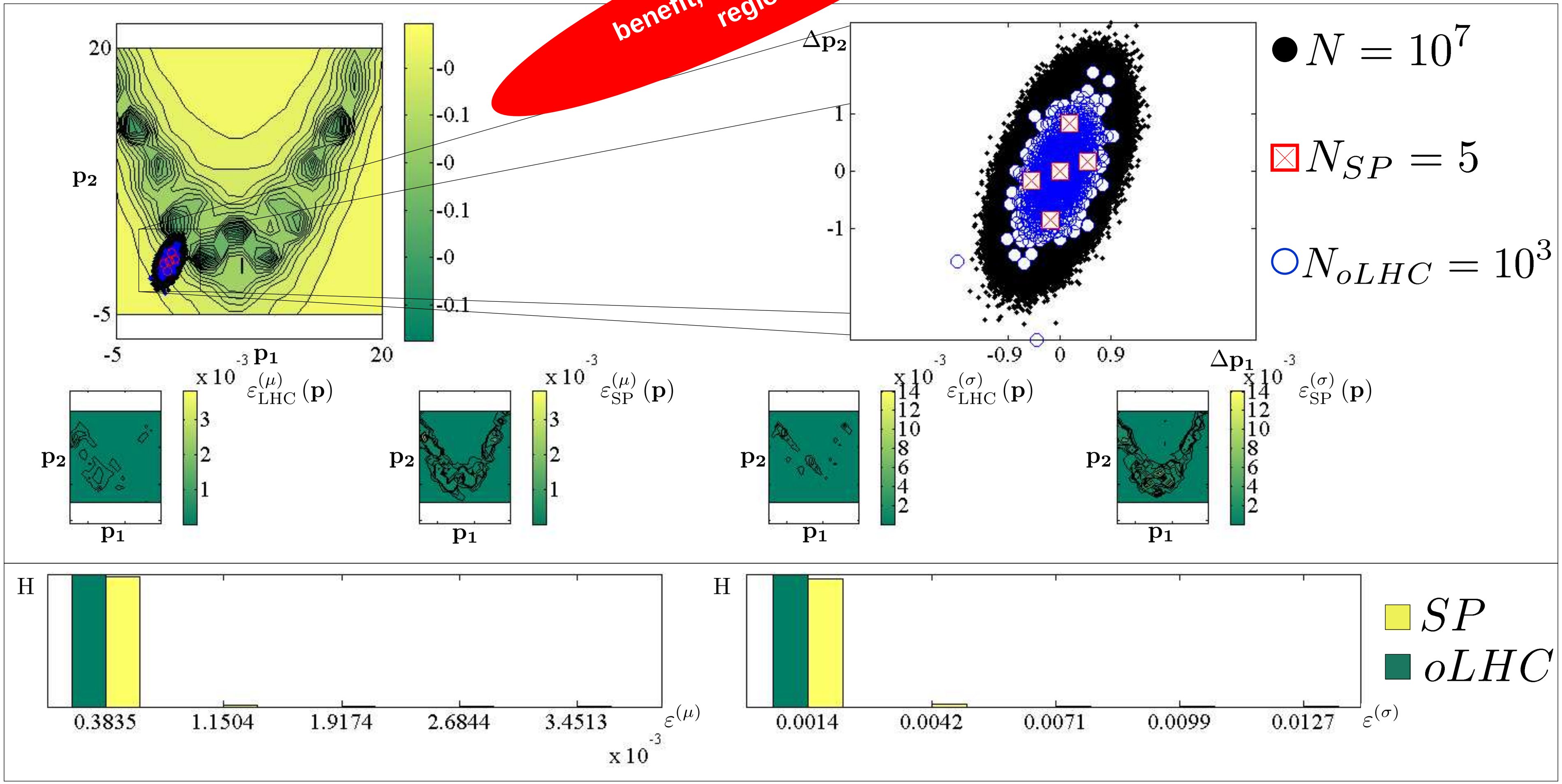


Performance assessment – SPM vs. oLHC

Problem : RCOS test function

Variability $\Delta \mathbf{p} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$

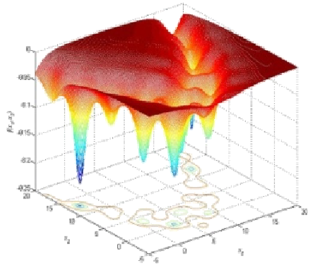
benefit, except for some regions!



How to use SPM for RDO?

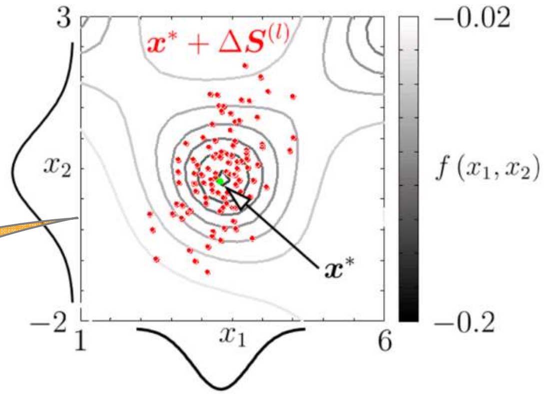
- design problem based on *Branin's rcos test function*

$$y(\mathbf{p}) = -\left[\left(p_2 - \left(5.1/4\pi^2\right)p_1^2 - 6\right)^2 + 10\left(1 + (8\pi)\right)\cos p_1 \cos p_2 + \log\left(p_1^2 + p_2^2 + 1\right) + 10\right]^{-1}$$



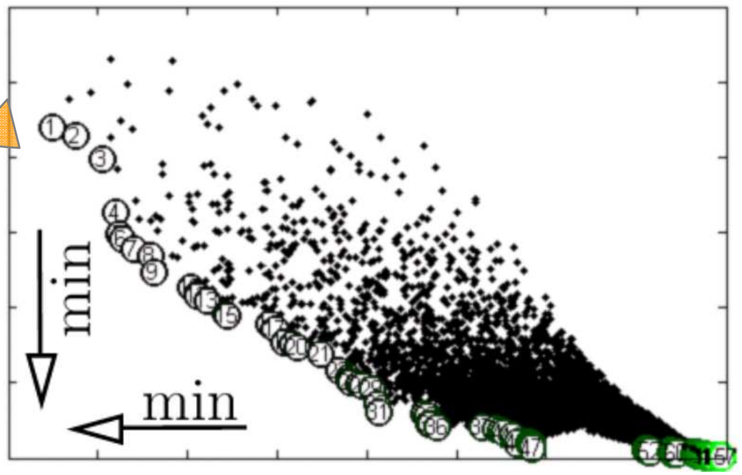
- input uncertainty: $\Delta\mathbf{p} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma})$

$$\boldsymbol{R} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \quad \boldsymbol{\sigma} = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix}$$



- robust design problem: $\min\limits_{-5 \le \mathbf{p} \le 20} \begin{bmatrix} \mu_Y(\mathbf{p}) \\ \sigma_Y(\mathbf{p}) \end{bmatrix}$

Is it possible to estimate prediction quality of SPM estimates?



- adjusted coefficient of determination

How to use SPM for RDO?

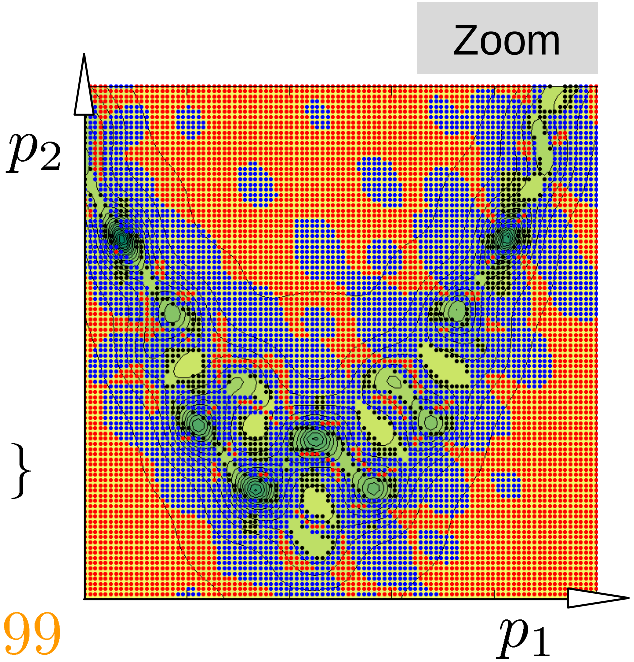
- influence of quality indicator constraint on feasible design space and results
  - Assumption: if transfer function is mostly linear around mean value, SPM will estimate exact results.
  - Do linear regression and compute adjusted coefficient of determination.
    → *The higher COD the more accurate are SPM results.*

$$\hat{f}_R(\mathbf{p},\mathbf{b}) = b_0 + \sum_{j=1}^{n} b_j p_j \rightarrow R^2_{adj}$$

  - extended RDO:

$$\min_{\mathbf{p}\in\mathcal{P}} \begin{bmatrix} \mu_Y(\mathbf{p}) \\ \sigma_Y(\mathbf{p}) \end{bmatrix} \text{ where}$$

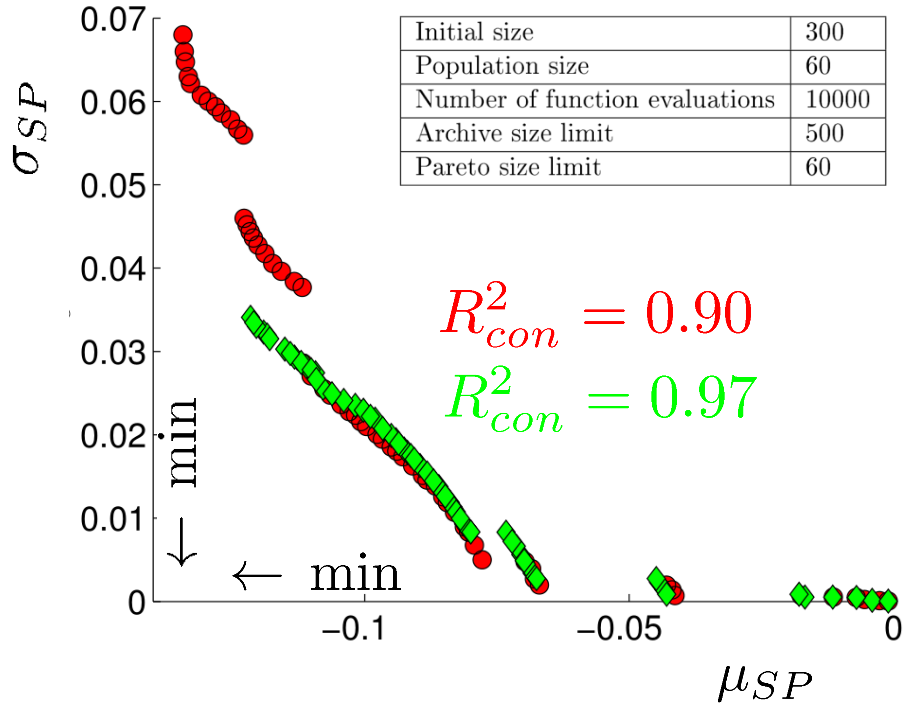
$$\mathcal{P} = \{\, \mathbf{p} \in \mathbb{R}^2 \,|\, -\mathbf{5} \le \mathbf{p} \le \mathbf{20}\,,\ R^2_{adj}(\mathbf{p}) \ge R^2_{con} \,\}$$

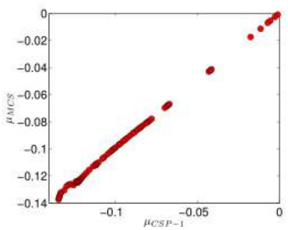


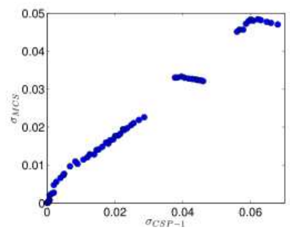
$R^2_{con} = 0.80$ $R^2_{con} = 0.90$ $R^2_{con} = 0.99$

How to use SPM for RDO?

- results of two different *AMGA* optimisation runs
- validation of non-dominated designs using LHC with $N = 10^7$
- comparison by computation linear correlation coefficients between SPM and LHC values

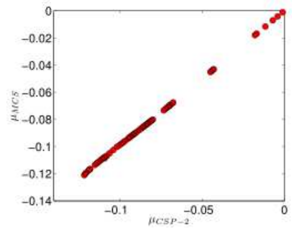


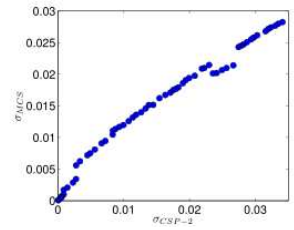




$\rho^{\mu}_{P} = 0.99$ $\rho^{\sigma}_{P} = 1.0$

$\rho^{\mu}_{S} = 0.98$ $\rho^{\sigma}_{S} = 0.98$





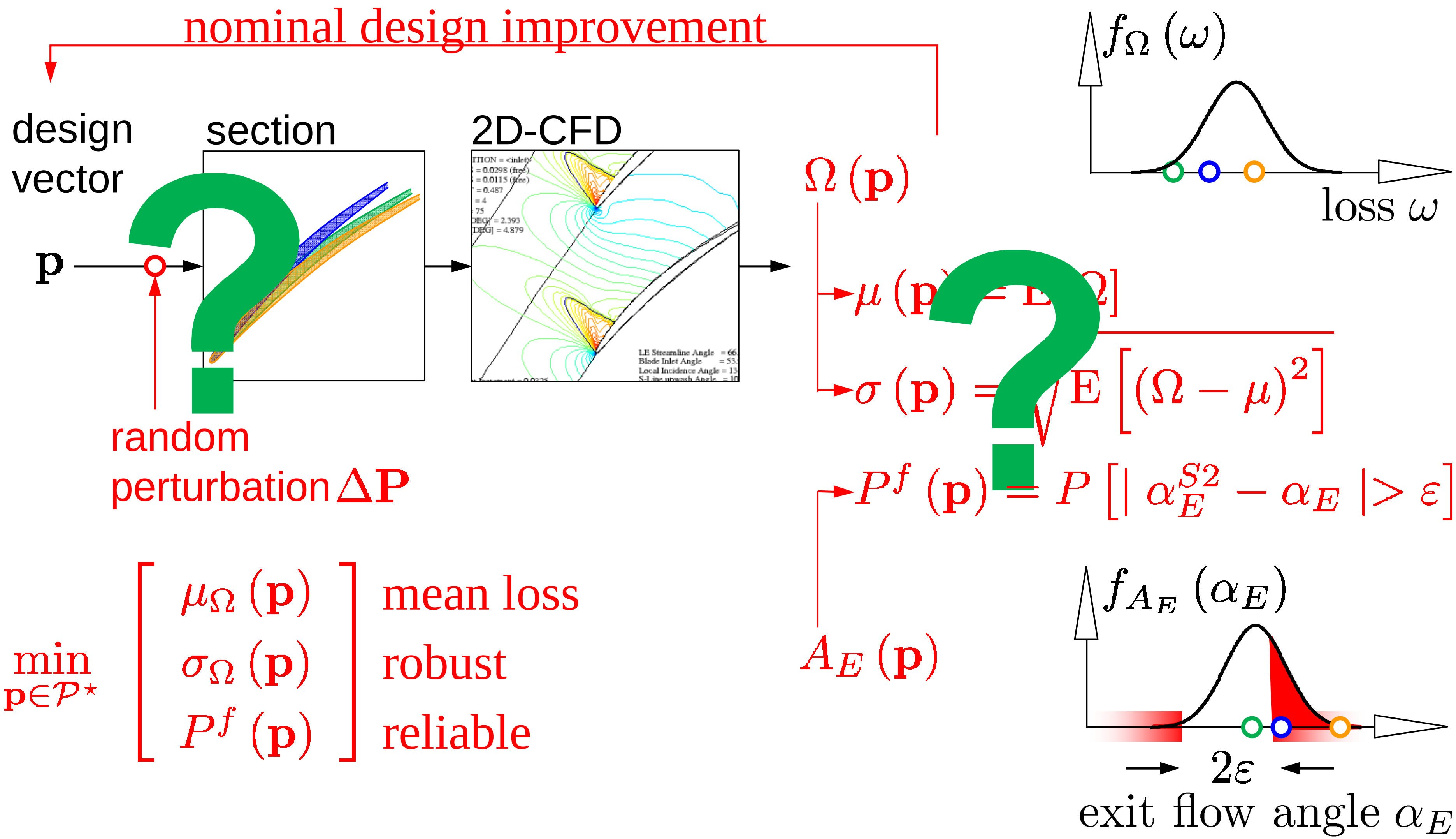
$\rho^{\mu}_{P} = 0.99$ $\rho^{\sigma}_{P} = 1.0$

$\rho^{\mu}_{S} = 0.99$ $\rho^{\sigma}_{S} = 0.99$

eRDO enables excellent results w.r.t. order of designs in criterion space!

Robust aerodynamic compressor blade design

- aerodynamic 2D robust design problem:



Robust aerodynamic compressor blade design

- quantification of manufacturing uncertainties
  - surface measurements of 147 manufactured blades by TU Dresden

  data reduction & identification

  - surface parametric model according to *Lange u.a. (2009)*

$$\mathbf{d} = \left[\zeta_d \;\; c_d \;\; f^{(max)} \;\; x_{d,f} \;\; T^{(max)} \;\; p_d \;\; T_I \;\; x_{d,I} \;\; T_E \;\; x_{d,E}\right]^{\mathrm{T}} \in \mathbb{R}^{10}$$

  statistical evaluation

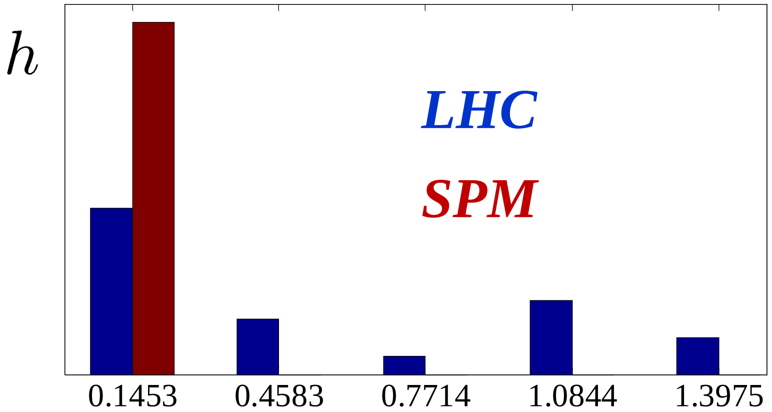
$$\Delta\mathbf{D} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}) \qquad \mathbf{\Sigma} \in \mathbb{R}^{10\times 10}$$

- design vector $\mathbf{p} \in \mathbb{R}^{14}$ differs from the uncertain vector $\mathbf{d} \in \mathbb{R}^{10}$

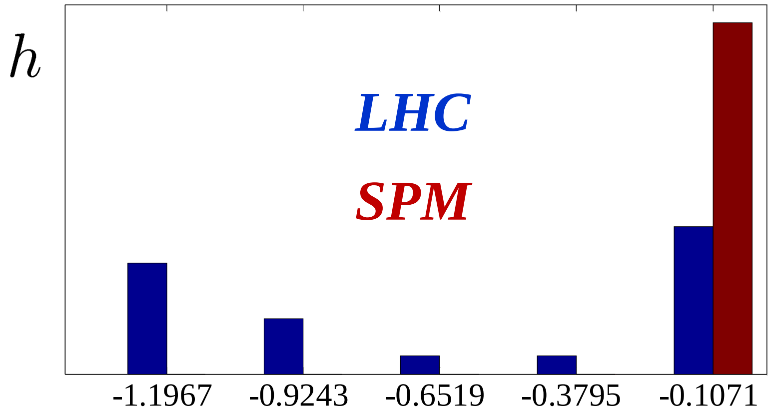
- test of SP method by performing DoE in $\mathbf{p}$

Robust aerodynamic compressor blade design

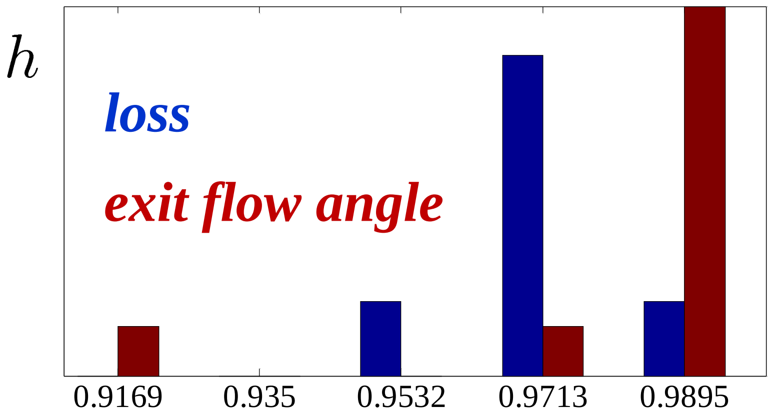
- DoE results: SPM vs. LHC of equal size, N=21
- compute error values based on LHC with N=200



$$\varepsilon_\omega^\mu = \mu_\omega^{val} - \mu_\omega \; [\%]$$



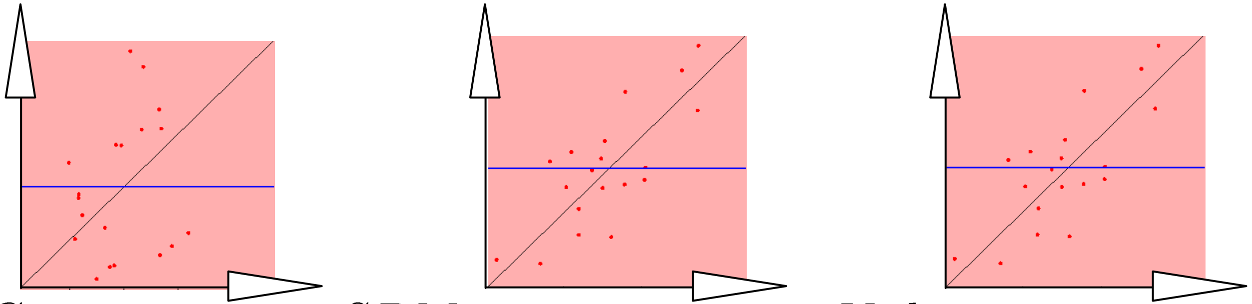
$$\varepsilon_\omega^\sigma = \sigma_\omega^{val} - \sigma_\omega \; [\%]$$



R^2_{adj}

General industrial applicability

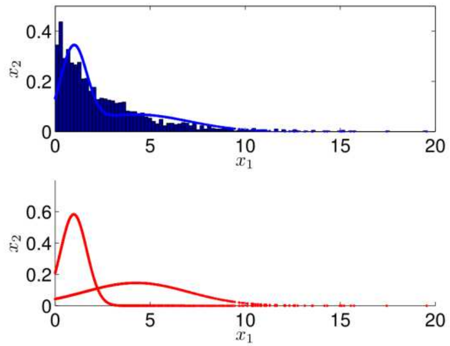
- Does prediction capability of a RSM increases due to deterministic property of SPM? –Cross Validation loss RBF Models



$\rho^{LHC} = 0.26$ $\rho^{SPM} = 0.57$ $\rho^{Val} = 0.57$

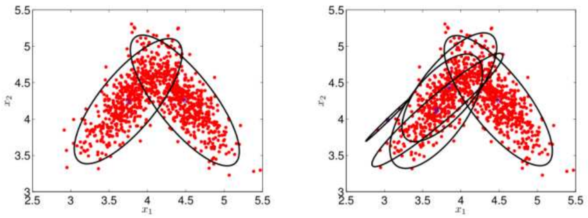
Yes it does!

- Is it possible to also model arbitrary variability?



  – *SPM* & mixture of Gaussians in conjunction with cluster and expectation maximisation algorithm $\rightarrow$ prediction accuracy increases due to more points

  – transformation into Gaussian space by e.g. Cholesky Decomposition or Rosenblatt transformation



Yes it is!

Conclusions

- compared to MCS the SPM provides same accuracy with less effort or better accuracy with some effort
- adjusted coefficient of determination can be used as quality indicator
- application of extended RDO to analytical test function using SPM provides excellent results from industrial point of view

Outlook

- application to robust design optimisation of compressor blades
- detailed analysis of SPM in the presence of non Gaussian input variability

References

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Comparison SP vs. optimal-LHC

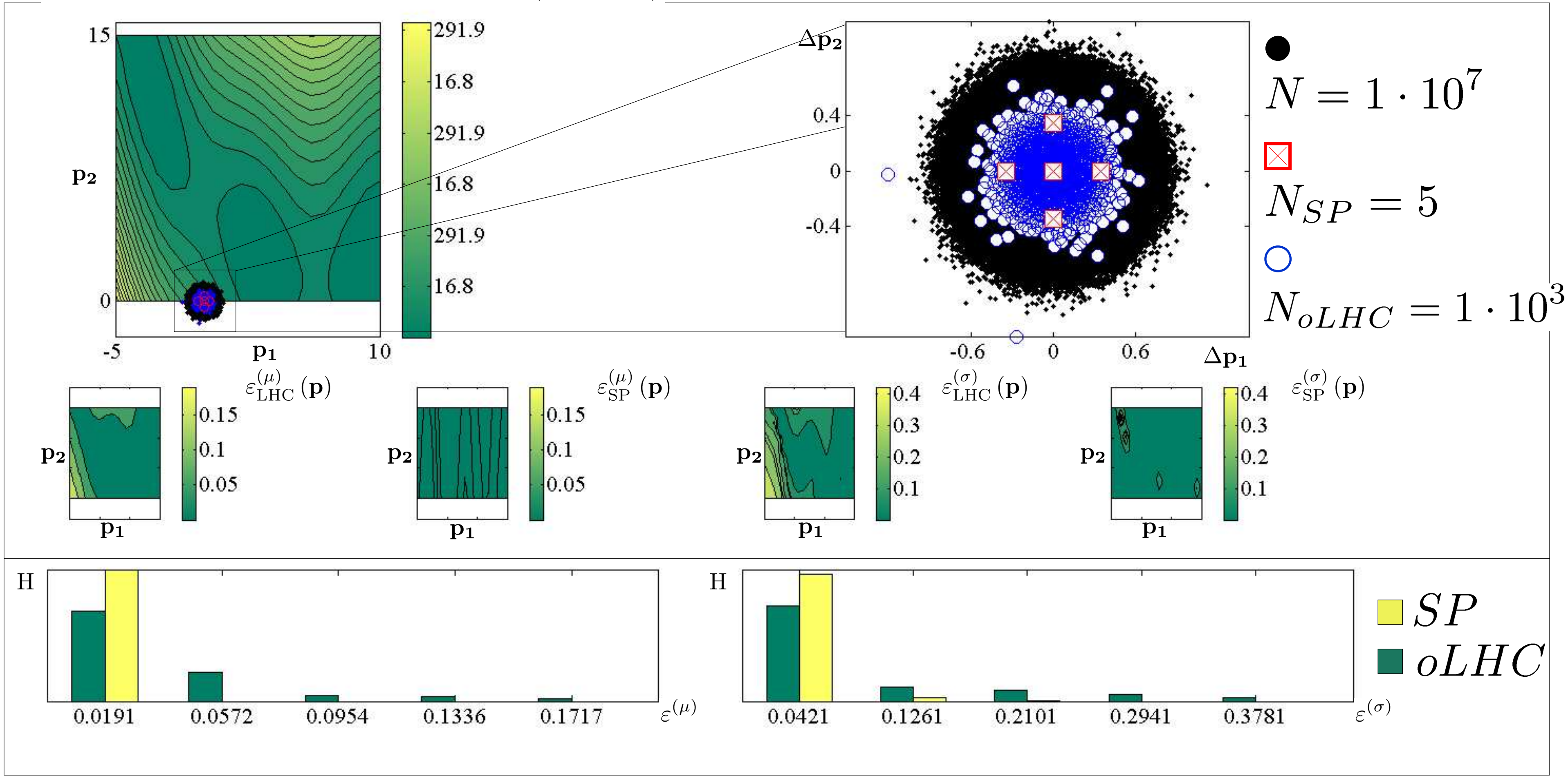
$$f(\mathbf{p}) = \left(p_2 - \left(51 \,/\, \left(40 * \pi^2\right)\right) * p_1^2 + (5/\pi) * p_1 - 6\right)^2 + 10 * \left(1 - \left(1 \,/\, (8 * \pi)\right)\right) * \cos(p_1) + 10$$

where $\mathbf{p} \in \mathbb{R}^2$

and $-5 \leq p_1 \leq 10$

and $-0 \leq p_2 \leq 15$

$\textbf{Variability}: \Delta\mathbf{p_1} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma_1})$



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