Robust Design Optimization of a Centrifugal Compressor **ANSYS**[®] concerning Fluid-Structure Interaction and Manyfacturing Tolerances W Th optimizing structural language

Dirk Roos DYNARDO GmbH dirk.roos@dynardo.de Johannes Einzinger ANSYS Continental Europe johannes.einzinger@ansys.com

Motivation

ANSYS[®] dunando



Outline

- Workbench Platform
- Parameterization of the Geometry
- Parameterization of the CFD Simulation
- Parameterization of the Mechanical Simulation
- Parametric Process Integration
- Sensitivity Analysis
- Design Optimization
- Robustness Evaluation
- Random Fields
- Robust Design Optimization
- Reliability Analysis







Workbench Parametric Process



ANSYS[®]

Workbench Platform & optiSLang





ANSYS Workbench

Structural Mechanics - Fluid Dynamics - Heat Transfer - Electromagnetic









A Multi-Physics Design and Analysis System







CFD Simulation

Mechanical Simulation

AProcess Integration

Colluization

Conformal Mapping





8

Conformal Mapping





BladeModeler

ANSYS® dynando

- Blade Design abilities in DesignModeler
- Angle/Thickness modifications in BladeEditor
- Multi-Stage Machines



Meridian Contour



M A: Compressor Example - DesignModeler		De	tails View	
File Create Concent Tools View Help			Details of FlowPa	thCompressor
Image Control of the second sec			Flow Path	FlowPathCompressor
MeridianPlane - 🛧 TEContour - ಶ 🦻	Generate 🖤 Share Topology 🛛 🕞 Extrude 🏤 Revolve 🌜 Sweep 🚯 Skin/Loft 🔲 Thin/Surface 💊 Blend		Machine Type	Centrifugal Compressor
BladeEditor: 🖓 Import BGD / BLoad BGD 🛛 😫 FlowPath	🔍 Cam ThkDef 🥒 Blade 🛷 Splitter 🚽 Vista TFExport 📐 Export Points 🎟 StageFluid Zone 媛 Throat Area		Theta Direction	Right Handed
Tree Outline 4	Graphics		Hub Coptour	HubContour
A: Compressor Example			Shroud Contour	ShroudContour
ZXPlane				
			Inlet Contour	InietContour
HubContour			Outlet Contour	OutletContour
InletContour			Hub Cut?	No
OutletContour			Shroud Cut?	No
LEContour			Number of Layers	5
ーー・ Contour ーー・ C型 RVLE	ExitWidth 🛶 🚽		Layer Details: 1	
			Layer Type	Fixed Span
🖓 O Parts, O Bodies			Span Fraction 1	0
			Laver Details: 2	
Sketching Modeling			Laver Tune	Fixed Span
Details View 4				
Plane MeridianPlane			Span Fraction 2	0.25
Sketches 9	////		Layer Details: 3	
Type From Plane Base Plane XYPlane	////		Layer Type	Fixed Span
Transform 1 (RMB) Rotate about Global Y			Span Fraction 3	0.5
FD1, Value 1 -90 ° Transform 2 (RMB) Rotate about Global 7			Layer Details: 4	
□ FD2, Value 2 -90 °	<mark>, , , , , , , , , , , , , , , , , , , </mark>		Layer Type	Fixed Span
Transform 3 (RMB) None Reverse Normal/7-Axis? No			Span Fraction 4	0.75
Flip XY-Axes? No	InterWidth g		Layer Details: 5	
Export Coordinate System? No			Laver Type	Fixed Span
			Span Fraction 5	1
	0.00 100.08			
	50.00		150.00	
	50.00		150.00	
	Model View Print Preview			
🥝 Ready	1 Plane			Millimeter 0 0

Impeller Hub, CamThk



	Details View 4		Angle: CamThkDefImpellerHub (Beta, Laver 1)	ф.
🚥 A: Compressor Example - DesignModeler	Details of CamThkDefImpellerHu	Ь		
File Create Concept Tools View Help	Camberline/Thickness	CamThkDefImpellerHub	3.36	
🛛 🔄 🔚 🖾 🗍 💬 Undo 📿 Redo 🛛 Select: 🆎 🍢	Flow Path			
MeridianPlane 🔻 株 TEContour 💌 ಶ 😔 Genera	Laver Number	1	<i>ν</i> το σο	
BladeEditor: 🚜 Import BGD 🖉 Load BGD 🚍 FlowPath 🚿 🔍	Details of Comborline	1	⊕ -10,00	
Select Laver: ElowPathCompressor V Laver 1 : CamThkDefImpe		V		
Tree Outline		res	C 9-20.00	
E	Angle Definition Type	Beta	8 -30.00	
XYPlane	Theta Reference	Leading Edge	P i	
XPlane	FD1, Theta at Reference(degree)	0	₹ -40.00	
	Angle Data Location	Camberline		
ElowPathCompressor	Details of Thickness		-51.36	
	Thickness Definition	Yes	0.00 12.50 25.00 37.50 50.00 62.50	75.00 100.00
The parts, o bodies	Thickness Definition Type	Normal to Camberline on Layer Surf	% M-Prime (LE to TE	E)
	Details of Angle Point 1			,
	D FD1001, Y of Angle Point 1	-48	Thickness: CamThkDefImpellerHub (Normal to Camberline on La	ver Surface, Laver 9
	Details of Angle Point 2			
	X of Angle Point 2	40	6.42	
Chatables as a s	D ED1003. Y of Apple Point 2	-25	••	
	Details of Angle Point 2			
Details View P	V of Apple Point 3	70	5.00	
Details of CamThkDefImpellerHub ComThkDefImpellerHub	ED1005 Victoria Deiek 0	70		
Elow Path ElowPath	D FD1005, Y or Angle Point 3	-25		
Layer Number 1	Details of Angle Point 4			
Details of Camberline	D FD1007, Y of Angle Point 4	-25		
Angle Definition Yes	Details of Thickness Point 1		≗3.00	
Angle Definition Type Beta	D FD1009, Y of Thickness Point 1	1		
Theta Reference Leading Edge	Details of Thickness Point 2		Ë 2.00 −	
Angle Data Location Camberline	X of Thickness Point 2	10		
Details of Thickness	D FD1011, Y of Thickness Point 2	6	1.00	
Thickness Definition Yes	Details of Thickness Point 3			
Thickness Definition Type Normal to Camberline	D ED1013 V of Thickness Point 3	6		
Details of Angle Point 1 Sproot Hugh hade Details 1	Details of Thickness Point 4	0	0.42	
Details of Angle Point 2	V of Thickness Point 4	20	0.00E+0 2.50E+1 5.00E+1 7.	.50E+1 1.00E+2
X of Angle Point 2 40		20		
D ED1003 V of Angle Point 2 -25	D FD1015, Y of Thickness Point 4	6	% ™ (LE to TE)	
🕜 Ready			J	

Impeller Shroud, CamThk



M A: Compressor Example - DesignModeler	Details View	4	Angle: CamThkDefImpellerShd (Beta, Layer 5)
Conversion Example - Designmodeter	Details of CamThkDefImpellerSl	nd	
File Create Concept Tools View Help	Camberline/Thickness	CamThkDefImpellerShd	3.85
🛛 🛃 🔚 🖾 🗍 🏵 Undo 📿 Redo 🗍 Select: 🆎 🏹	Flow Path	FlowPathCompressor	
MeridianPlane 💌 🛧 TEContour 🔍 💆 🛛 🧚 Generat	Laver Number	5	ន្ល -10.00 –
BladeEditor: 🖓 Import BGD 🛛 🗐 Load BGD 🛛 🚍 FlowPath 🔍 Ca	 Details of Camberline 	-	Ŭ
Select Layer: FlowPathCompressor 🔻 Layer 5 : CamThkDefImpelle	Angle Definition	Yes	Ψ ^{°-20,00}
Tree Outline 7	Angle Definition Type	Beta	.⊆ -30.00
E	Theta Reference	Leading Edge	
XYPlane	ED1 Theta at Beference(degree)		£-40.00
	Ande Data Leastice	Constanting	S 50.00
	Angle Data Location	Camberline	-50,00
FlowPathCompressor	Details of Thickness		-58.85
	Thickness Definition	Yes	0.00 12.50 25.00 37.50 50.00 62.50 75.00 100.0
CaminkDerImpelersno	Thickness Definition Type	Normal to Camberline on Layer Surf	% M-Prime (LE to TE)
	Details of Angle Point 1		
	D FD1001, Y of Angle Point 1	-55	Thickness: CamThkDefImpellerShd (Normal to Camberline on Laver Surface, Laver 5)
	Details of Angle Point 2		mickless, cairmice angelerand (vormal to camberline on cayer burace, cayer by
	X of Angle Point 2	44.083471	6.42
	D ED1003. Y of Angle Point 2	-45	••
Sketching Modeling	Details of Angle Point 3		
Details View 4		66 511139	5.00
Details of CamThkDefImpellerShd	D ED1005 V of Apple Point 3	-20	2.00 2.00
Camberline/Thickness CamThkDefImpellerShd		-50	Ŭ V
Flow Path FlowPathCompressor	- Decails of Angle Point 4		<u>, 5</u> 4.00 − − − − − − − − − − − − − − − − − −
Layer Number 5	D FD1007, Y of Angle Point 4	-30	
Details of Camberline	Details of Thickness Point 1		₩ 3.00 + -
Angle Definition Yes	X of Thickness Point 1	10	l ĝ
Theta Reference	D FD1009, Y of Thickness Point 1	6	
FD1, Theta at Reference(degree) 0	Details of Thickness Point 2		
Angle Data Location Camberline	X of Thickness Point 2	20	Ž
Details of Thickness	D ED1011. Y of Thickness Point 2	6	1.00
Thickness Definition Yes	Details of Thickness Point 2		
Thickness Definition Type Normal to Camberline	Details of Thickness Point 3	4	
Details of Angle Point 1	D FD1013, Y of Thickness Point 3	6	-0.42
D FD1001, Y of Angle Point 1 -55	Details of Thickness Point 4		0.00E+0 2.50E+1 5.00E+1 7.50E+1 9.71E
X of Angle Point 2 44.083471	D FD1015, Y of Thickness Point 4	1	% M (LE to TE)
■ FD1003_V of Anale Point 2 -45			
🥝 Ready		No Selection	Millimeter 0 0 //

Impeller Blade



A: Compressor Example	- DesignModeler						
File Create Concept Tools	File Create Concent Tools Weiw Help						
	do @Redo Select: * 5			150 1 6 13			
						- Pland - Chamfey @ Doint PPI	
MeridianPiane 🔻 🏊 i Eu	Contour 💌 🔑 🛛 🥜 Genera	ate (myonare ropology) in the second	xtrude Bakevoive @soweep	SkinyLore in the	Inface	Biena + to Chamrer to Point in it	Parameters
BladeEditor: 🎇 Import BGD	ELoad BGD 🛛 🔁 FlowPath 🔌 🔾	CamThkDet 💋 Blade 💋 Splitte	er 🚽 VistaTFExport 🔨 ExportPol	ints IIIIIStageFluidZone	- Kir	ThroatArea 🕢 Preferences	
Select Layer: FlowPathCompres	ssor : 🔻 Layer 1 : CamThkDefImpel	illerHut 🔻					
Tree Outline	P	Graphics				Angle: CamThkDefImpellerHub (Be	ta, Layer 1, Impeller) 4
A: Compressor Example	le				NS	SYS	
VZPlane						₩ _{-10.00}	
⊕							<u> </u>
CamThkDefImpel	llerHub		P1+1	F III		2-20,00	
	lerShd				De	tails View	4
Impeller						Details of Impeller	
Harts, 20 Dour	les						- u
						Blade	Impeller
						Camberline Definitions	2
Sketching Modeling						Туре	Rotor
Details View	₽					ED1 Number of Blade Sets	20
Details of Impeller						C for Contruction	20 2
Blade	Impeller					Surface Construction	General
Camberline Definitions	2	Intervvid				Blade Extension (%)	2
Type	Rotor					Leading Edge Details	
Surface Construction	; 20 General					LECoptour	LECoptour
Blade Extension (%)	2						
Leading Edge Details						Туре	Ellipse
LEContour	LEContour					FD2, LE Ratio at Hub	3
Туре	Ellipse					ED3. LE Ratio at Shroud	3
FD2, LE Ratio at Hub	3					Turiling Edge Details	
Trailing Edge Details	3		/			Trailing coge vecalis	
TEContour	TEContour					TEContour	TEContour
Туре	Cut Off		0.00 91			Туре	Cut Off
🖃 Camberline/Thickness Def	initions: 2					Camberline/Thickness Defi	nitions: 2
CamberThick Def. 1	CamThkDefImpellerHub		45.00			Combernie, menness ser	
CamberThick Def. 2	CamThkDefImpellerShd						CaminkDerImpelierHub
		Model View Print Preview				CamberThick Def. 2	CamThkDefImpellerShd
🥝 Ready				No Select		1	

© 2010 ANSYS, Inc. All rights reserved.

Return Vane Hub, CamThk



Di	A: Compressor Example - Desi	gnModeler	De	eta	
	File Create Concept Tools View	Help	Ξ	D	
,	🕅 🔲 🗒 🔯 🗍 Dundo 🔅	Redo Select: *[c	
MeridianPlane					
10				L	
] > 	elect Layer: HowPathCompressor 🔻	Layer 1 : CamThkD	E	D	
	ee Outline			F.	
1				-	
	ZXPlane			A	
	VZPlane			т	
	FlowPathCompressor			F	
	CamThkDefImpellerHub			Ľ	
	CamThkDefImpellerShd			A	
				D	
	🛨 💦 20 Parts, 20 Bodies	\searrow		Ē	
				브	
				T	
Sketching Modeling					
Details View					
Details of CamThkDefR¥Hub					
	Camberline/Thickness	CamThkDefRVHub		Г	
	Flow Path	FlowPathCompressor			
	Layer Number	1		-	
	Details of Lamberline	u			
	Angle Definition	Yes		n	
	Angle Definition Type	beta Leading Edge		Ľ	
	ED1 Theta at Reference(degree)	Ceauling Luge			
	Angle Data Location	Camberline		D	
	Details of Thickness				
	Thickness Definition	Yes			
	Thickness Definition Type	Normal to Camberline			
Ξ	Details of Angle Point 1	-			
	X of Angle Point 1	60		Ľ	
	Y of Angle Point 1	0		Г	
-	Details of Angle Point 2				
	Y of Angle Point 2	0		Ľ	
E	Details of Angle Point 3				
(🕗 Readv			_	

etails View	4			
Details of CamThkDefR¥Hub				
Camberline/Thickness	CamThkDefRVHub			
Flow Path	FlowPathCompressor			
Layer Number	1			
Details of Camberline				
Angle Definition	Yes			
Angle Definition Type	Beta			
Theta Reference	Leading Edge			
FD1, Theta at Reference(degree)	0			
Angle Data Location	Camberline			
Details of Thickness				
Thickness Definition	Yes			
Thickness Definition Type	Normal to Camberline on			
Details of Angle Point 1				
X of Angle Point 1	60			
Y of Angle Point 1	0			
Details of Angle Point 2				
Y of Angle Point 2	0			
Details of Angle Point 3				
D FD1005, Y of Angle Point 3	60			
Details of Thickness Point 1				
X of Thickness Point 1	40			
D FD1007, Y of Thickness Point 1	45			
Details of Thickness Point 3				
Y of Thickness Point 3	6			
Details of Thickness Point 4				
Y of Thickness Point 4	10			





Return Vane Shroud, CamThk



98.85

🕅 A: Compressor Example -	DesignModeler	D	etails View	P	Angle: CamThkDefRVShd (Beta, Layer 5)
File Create Concept Tools V	view Help	E	Details of CamThkDefR¥Shd		
- 🔄 🛃 📕 📫 🗍 🏵 Undo	🕜 🕜 Redo 🛛 Select: 🏼 🔭		Camberline/Thickness	CamThkDefRVShd	64.20
MeridianPlane 🔻 抺 TECc	ontour 👻 ಶ 🗸 🕫		Elow Path	FlowPathCompressor	9 50 00 -
BladeEditor: 🔏 Import BGD	Load BGD 📑 FlowPath		Laver Number	E	
Select Layer: FlowPathCompress	or 🔻 Layer 5 : CamThkDefi		Layer Number	2	€ 40.00
Tree Outline		E	Details of Camberline		.⊆ 30.00
E			Angle Definition	Yes	8 20.00
XYPlane			Angle Definition Type	Beta	
			Theta Reference	Leading Edge	₹ 15,00
ElowPathCompress	or		ED1 Theta at Deference/degree)	0	-4.20
CamThkDefImpeller	rHub		L FD1, Theta at Kererence(degree)	0	0.00 25.00 50.00 75.00
CamThkDefImpeller	rShd		Angle Data Location	Camberline	% M-Prime (LE to TE)
CamThkDefRVHub		E	Details of Thickness		% MEPTINE (LE 10 TE)
CamThkDefRVShd			Thickness Definition	Yes	
🛨 🦳 🖓 20 Parts, 20 Bodies	s		Thickness Definition Type	Normal to Cambersurface	Thickness: CamThkDetRVShd (Normal to Cambersurface, Layer 5)
Skotsbing			Details of Angle Point 1		10.15
				<u>(0</u>	48.15
Details View			D FD1001, Y or Angle Point 1	60	
Camberline/Thickness	CamThkDefRVShd	E	Details of Angle Point 2		v 40.00
Flow Path	FlowPathCompressor		X of Angle Point 2	60	<u>e</u>
Layer Number	5		V of Apgle Point 2	0	× · · · · · · · · · · · · · · · · · · ·
Details of Camberline				0	Ê ^{30,00} //
Angle Definition	Yes	E	Details of Angle Point 3		
Angle Definition Type	Beta		Y of Angle Point 3	0	
ED1. Theta at Reference(der	aree) 0		Details of Thickness Point 1		g 20.00
Angle Data Location	Camberline		Decails of Thickness Foline 1		ि ल 🚺
Details of Thickness			Y of Thickness Point 1	10	
Thickness Definition	Yes		Details of Thickness Point 2		² 10.00 −
Thickness Definition Type	Normal to Cambersurf		V of Thickness Daiph 2	40	2
Details of Angle Point 1				40	
D FD1001, Y of Angle Point 1	60		D FD1009, Y of Thickness Point 2	45	2.15
Details of Angle Point 2			Details of Thickness Point 3		
X or Angle Point 2	60			-	0.00E+0 2.30E+1 5.00E+1 /.50E+1
Details of Angle Point 2	U		Y of Thickness Point 3	6	% M (I E to TE)
🍼 Ready					



1.00E+2

Return Vane Blade



M A: Compressor Example	- DesignModeler		D	etails View	
File Create Concept Tools	View Heln		E	Details of Return¥ane	
) 🖉 📑 📑 🚳 🛛 🌒 und	lo @Redo Select: *	- K K K B & S + C + C + C + K +	6	Blade	ReturnVane
MeridianPlane 🔻 🗚 TEC	Contour 👻 🗾 🗲 Gene.	rate 💖 Share Topology 🛛 🔀 Extrude 🏘 Revolve 🌜 Sweep 🚯 Skin/Loft 🕽	(Camberline Definitions	2
BladeEditor: 🔏 Import BGD 🛔	🖺 Load BGD 🛛 😫 FlowPath 🚿	, CamThkDef 🖉 Blade 🥏 Splitter 🚽 VistaTFExport 📉 ExportPoints 🎟 Stag	jeFlu	Туре	Stator
Select Layer: FlowPathCompres	ssor : ▼ Layer 1 : CamThkDefRVH			ED1, Number of Blade Sets	24
				Surface Construction	General
			L	Blade Extension (%)	2
FlowPathCompres	ssor erHub		E	Leading Edge Details	
	'erShd			LEContour	RVLE
CamThkDefRVHub	5			Туре	Ellipse
Employed CamThkDefRVShc				E FD2, LE Ratio at Hub	1
🗄 🗸 🐨 44 Parts, 44 Bodik	es	Extra	1	EFD3, LE Ratio at Shroud	1
Sketching Modeling				Trailing Edge Details	
Details View	Ļ			TEContour	RVTE
Details of Return¥ane				T	
Blade	ReturnVane			туре	Ellipse
Camberline Definitions	2			FD4, TE Ratio at Hub	1
Туре	Stator				-
FD1, Number of Blade Sets	24			E FD5, TE Ratio at Shroud	1
Surface Construction	General		F	Camberline/Thickness Defi	nitions: 2
Blade Extension (%)	2			Contracting of the	Courth Definition
E Leading Edge Details	DUIE		-	Camper Thick Def. 1	CaminkDerRVHub
Tupe	Filippo			CamberThick Def. 2	CamThkDefRVShd
ED2 LE Distion at Link	1				
ED3 LE Ratio at Flue	1			🛉 20.00 +	
Trailing Edge Details	<u> </u> *				
TEContour	RVTE			E 10 00	
Туре	Ellipse			2 10.00	
FD4, TE Ratio at Hub	1			Z Z	T I
FD5, TE Ratio at Shroud	1	0.00 			
Camberline/Thickness Defi	initions: 2			-3.15	50E+1 1 25E+2 2 0.005+2
CamberThick Def. 1	CamThkDefRVHub	50.00		1.21E+1 /.	JUC+1 1.2JE+2 2.08E+2
CamberThick Def. 2	CamThkDefRVShd				M (LE to TE)
		Model View Print Preview			
🧭 Ready			No Sele	ection	Millimeter 0 0

Geometry Export, TurboGrid



🔯 A: Compressor Example - DesignModeler			
File Create Concept Tools View Help			
MeridianPlane 🔹 🗚 TEContour 🔹 🏂 🧳 Generate 🐄 Share Topology 🕞 Extrude 🏤 Revolve 🏡 Sweep 🚯 Skin/Loft 💼 Thin/Surf	rface	💊 Blend 👻 🔦 Chamfer 🛭 🛷 Point 🛛 🔀 Paran	neters
BladeEditor: 🍪 Import BGD 🕼 Load BGD 🗮 FlowPath 🚿 CamThkDef 🥒 Blade 🛷 Splitter 🚽 VistaTFExport 🍾 ExportPoints 🎟 StageFluidZone	V≪⊺t	hroatArea APreferences	
Select Layer: FlowPathCompressor V Layer 1 : CamThkDefImpellerHut V	D	etails View	l
Tree Outline Graphics Graphics		Details of ExportPointsPetu	IFD Y 2D 8
		Supert Deints	
FlowPathCompressor		Export Points	ExportPointsReturnvane
Cam ThkDef Impeller Hub		Export Type	TurboGrid
		Export to file	No
CamThkDefRVHub		Base Feature	FlowPathCompressor
CamThKDerRyShd ReturnVane		Blade Info From	User Specified
VistaTFExport		FD1, Number of Blades	24
ExportPointsReturnVane		FD2, Blade Row Number	2
🕀 🗸 🚯 44 Parts, 44 Bodies		Flow Path	FlowPathCompressor
Sketching Modeling		Blade Surfaces	4
Details View 4		ED3. Hub/Shroud Offset %	0.5
Details of ExportPointsReturnVane			0.1
Export Points ExportPoints			0.1
Export to file No		Layer: I	1
Base Feature FlowPathCo		Output?	Yes
Blade Info From User Specified	E	Layer: 2	
FD1, Number of Blades 24		Output?	Yes
Flow Path FlowPath FlowPathCo.		Laver: 3	
Blade Surfaces 4	- I-	Output?	Vec
FD3, Hub/Shroud Offset % 0.5			165
FD4, Point Tolerance 0.1	7	Layer: 4	
Layer: 1 Output Vec	L	Output?	Yes
□ Layer: 2 0,00 200,00 (mm) X	E	Layer: 5	
Output? Yes		Output?	Yes
E Layer: 3		· ·	1
Output? Yes Model View Print Preview			
No Selection	חר	,	Millimeter 0 0

3D Blade Design





Geometry of Impeller

ANSYS[®]



One sector Model: A, no blend B, blend 1 mm





© 2010 ANSYS, Inc. All rights reserved.

Geometry

CFD Simulation

Mechanical Simulation

Process Integration

Columization

ANSYS[®]

dunando

Vista CCD, Overview

ANSYS° dynando

- Preliminary Design
 - Mean line, or "1D"
 - Velocity triangles
 - Defines the geometrical envelope within the larger picture
 - Determines the performance potential
 - Errors very expensive to recover later
 - Semi-empirical approach
 - Many calculations, needs to be rapid and robust
 - Can be amenable to optimization



Vista TF, Overview



S2

- Expand into 2D
 - S2 or 'through flow' approach
 - Solution of the circumferentially averaged equations of motion
 - Take account of the span wise variation in design parameters
 - First estimates of 3D geometry can be made, both aero and mechanical aspects

Refine in 3D

- Accept longer run times
- Absolute accuracy important

Vista TF, Solution Procedure



- Approximate meridional streamlines by splitting the flow into regions of equal area and make a first guess of all parameters
 - s from loss correlations
 - h_t from Euler equation
 - $-~\rho$ from \textbf{h}_{t} and s
- Determine the distribution of c_m
 - Gradient of the meridional velocity from general radial equilibrium
 - Level of the meridional velocity from the continuity equation



- Adjust $c_{\rm m}$ on mean streamline until both gradient and mass flow satisfied
- Improve estimate of the streamline positions and recalculate
- Iterate until c_m and all other parameters are converged

Vista TF Simulation Result





TurboGrid Mesh, Impeller





TurboGrid Mesh, Return Vane





CFX Preprocessing











CFX Postprocessing



	Outline Variables Expressions Calculators Turbo
◎ C4 : CFX - CFD-Post	
File Edit Session Insert Tools Help	
😤 📽 🖳 🗿 🗃 🤊 🥲 🧭 Location 🗸 🤹 📓 豪 豪 🎸 🍌 🕃 涩 🗭 🔘 🖉 🙆	ma Angular velocicy myomega
Outline Variables Expressions Calculators Turbo	
View 1 Vi	
□ □ □ □ ↓ S1 Blade Dev Arge Contour 1	Triniet mass-lowAve(local lemperature in Str. Frame)@R1 Iniet
-]]‡ S1 Hub	PHI Two Vie The set of the lat
- 0.000e+001	
5500+001	alstep Accompace Time Step
4 Stitle-OT	
Besh Regions	
🖻 🗑 User Locations and Plots	
Contour 1	IIIYAIRK 207.1 [J KJ ~1 K ~1]
GN Default Transform	
V II 2 Derault Legend view I	
R ³ Meridional Legend	Pri mytower mytonega mytorque, r[rau]
	Pt myslohaD 1 mascelow (vade)@D1 Iolet
Turbo Surface 1	
	PT myalphaco massriowave(now Angle)@C1 Outlet
🕂 🖓 🏠 Title Page	(-v*Welocity in Str. Erame u+v*Welocity in Str. Erame u)
	weta ((ptratio^((mykappa-1)/mykappa))-1)/(Ttratio-1)
	w mysed (myselectic (myselectic (myselectic)) myselectic (myselectic))
Batter Constant State	$\sim \overline{C}$ mymassin 72.6 [kn s^-1]
5 mne+on	wywega 699.76 [radian s^-1]
I ULAI LIGSSUIG/AGUU/	nBladesR1 21
Details of Contour 1	nBlades51 23
Geometry Labels Render View	R omega Angular Velocity
Show Contour Bands	- 🙀 pout 2300000 [Pa]
	- 🙀 ptin 1724000 [Pa]
	R ptinlet massFlowAve(Total Pressure in Stn Frame)@R1 Inlet+mypref
Draw Mode Smooth Shading	massElowAve(Total Pressure in Sto Frame)@S1.Outlet+mypref
	ptratio ptoutet/ptinlet
Face Culling No Culling 0.100	a sstep Sequence Step
	Time
Comment Vie	🛛 🗖 🖟 tstep 10 [rad] /abs(myomega)

CFX, Best Practice





Geometry

CFD Simulation

Mechanical Simulation

Process Integration

Robust Design Colimization

ANSYS[®]

dynando

Structural Meshing





Mesh	#Nodes A	#Nodes B
1	10612	18911
2	54356	60103
3	236883	334242

Cyclic Symmetry Structures



 $\begin{pmatrix} M_{11} & \dots & M_{1n} \\ \dots & \dots & \dots \\ M_{n1} & \dots & M_{nn} \end{pmatrix} \begin{pmatrix} \ddot{u}_1 \\ \dots \\ \ddot{u}_n \end{pmatrix} + \begin{pmatrix} K_{11} & \dots & K_{1n} \\ \dots & \dots & \dots \\ K_{n1} & \dots & K_{nn} \end{pmatrix} \begin{pmatrix} u_1 \\ \dots \\ u_n \end{pmatrix} = \begin{pmatrix} f_1(t) \\ \dots \\ f_n(t) \end{pmatrix}$ ANSYS $M_n \ddot{U}_n + K_n U_n = F_n(t)$ ANSY CentirfugalCompressorFEM1--Static Structural (B5) .rfugalCompressorFEM1--Static

Boundary Conditions and Loads





Mechanical, Displacement





Mechanical, Stress





© 2010 ANSYS, Inc. All rights reserved.

ANSYS, Inc. Proprietary
Mechanical, Modal Analysis





© 2010 ANSYS, Inc. All rights reserved.

Mode Tracking





Geometry

CFD Simulation

Mechanical Simulation

Process Integration

Robust Design

Parametric Process



Parametric Process



ANSYS[®]

Vista TF vs. CFX, Characteristic



ANSYS[®]

Vista TF vs. CFX, Characteristic





Vista TF vs. CFX, Characteristic





Vista TF vs. CFX, Choke





Vista TF vs. CFX, Optimum





Vista TF vs. CFX, Stall





Campbell Diagram



© 2010 ANSYS, Inc. All rights reserved.

ANSYS[®]

Inanco

Campbell Diagram





Optimization Objective





Defined Operating Point: Mass Flow Rate 72.6 kg/s Rotational Velocity Ω=6644 rev/min Total Pressure Ratio π=1.35±0.01, Objective Maximal Efficiency η=max, Objective No Resonance: Ω≠ω_i

Parameter Manager



Output Parameters				🖃 Input	Parameters			
P33 n	nBladesR1	21		ťp	P1	InletWidth	53.1	
P34 n	nBladesS1	23		ťρ	P8	ExitWidth	26.2	
р ⊋ Р36 Т	Ttratio	1.1034	Pr	ιþ	P9	RImpeller	305.3	
P37 m	myMassFlow	72.6	kg s^-1	ιþ	P10	HubBeta1	-48.4	
P38 m	myeta	0.86738	-	ιþ	P11	HubBeta2	-25.5	
🖓 РЗ9 р	ptratio	1.3503	.1	ιþ	P12	HubBeta3	-25.6	
P40 m	myPower	-2.2634E+06	W	ťρ	P13	ShdBeta1	-55.7	
P41 m	myTorque	-3234.5	J	ťρ	P14	ShdBeta2	-45.7	
P42 m	myDeltaS	0.012506		ιþ	P15	ShdBeta3	-30.7	
P43 m	myalphaRS	62.32	degree	ťρ	P16	HubThk1	1.1	
P44 n	myalphaR1	1.4839	degree	ťp	P17	HubThk2	6.2	
P45 rr	myalphaS1	19.598	degree	ťρ	P18	ShdThk1	1.1	
16 (p P21	RVHubThk1 45	.5		ťp	P19	ShdThk2	6.1	
17 G P22 18 G P23	RVHubBeta1 60. RVShdBeta1 60.	.5		ſp	P21	RVHubThk1	45.5	
19	RVShdThk1 45 ImpellerBlades 21	.5		űp.	P22	RVHubBeta1	60.5	
21 b P26	RVBlades 23 Ttin 31:	3 К 🔻		űp.	P23	RVShdBeta1	60.5	
23 b P28	myAirCP 10	04.4 J kg^-1 K^-1 ▼		¢,	P24	RVShdThk1	45.5	
25 b P30	mymassin 72	.6 kg s^-1 V		(p	P25	ImpellerBlades	21	
26 tp P31 27 tp P35	ptin 1.7	9.76 radian s^-1 ▼ 724E+06 Pa ▼		űp.	P26	RVBlades	23	
Input Dor	omoto	r _ 26		¢,	P27	Ttin	313	к
Input Par	amele	= 20		űp.	P28	myAirCP	1004.4	J kg^−1 K^−1
Output Parameter -43			¢,	P29	myAirR	287.1	J kg^−1 K^−1	
		-+		ſp.	P30	mymassin	72.6	kg s^-1
Constraints =		= 68	3	ſþ.	P31	myomega	699.76	radian s^-1
				ſþ.	P35	ptin	1.724E+06	Pa

Workbench Interface to optiSLang

	AxialStageFSIblend - Workbench			
	File Edit View Tools Units Help			
	💾 New 🚰 Open 🚽 Save 🗟 Save As 🍣 Reconnect 🚿 Refresh Project 🦩 Update Project 🁔 Import 🌀 Proj	ect 🕜 Compact Mode 🛛 📿 optiPlug		
	Toolbox _ X Project Schematic			_ `
	Analysis Systems		a 🔅 🦳 ontiD	lua 🕴
	Electric (ANSYS) ▼ A ▼ B	▼ C ▼	e 🛛 🖊 🖊 🖓	iug
	S Fluid Flow (CFX) 1 00 Geometry 1 🛠 TurboGrid	1 🕖 CFX 1 🚺 Steady-State		
	🔮 Fluid Flow (FLUENT) 2 🔞 Geometry 🗸 🔶 2 🚱 Turbo Mesh 🗸 _	🖊 2 🍓 Setup 🗸 🖌 2 🦪 Engineering D		
	Harmonic Response (ANSYS) 3 A Parameters TurboGrid	3 😭 Solution 🗸 🗸 🖊 3 🛞 Geometry		
	Sinear Buckling (Samcef) (Beta) Geometry	4 🥪 Results 🗸 4 🧼 Model		
	Magnetostatic (ANSYS)	> 5 🖗 Parameters		
0		CFX 6 Mini Solution		
6		8 Darameters		Parameters
		Steady-State T	hermal (ANSYS)	Static Structural (ANSYS)
ſ	Write optimization project 🔹 🗸		· · /	
	Optimization problem 💙			
	Lower bounds (-) 10 % Upper bounds (+) 20 %			
	Update mode			
	Don't write files, show a warning message 🔹 🗸 🗸			
	Save results			
	Show W/B GUI during calculations	· · · · · · · · · · · · · · · · · · ·		
				_ ;
	OK Cancel	В		С
		Details		Progress
	View All / Customize			
	Ready			🚥 Hide Progress 🛛 斗 Show 31 Messages

Workbench Interface to optiSLang

SC optiSLang 3.0.1 powered by flowGuide			
Start		st flowGuide Project Manager	
Project manager		File	
Result monitoring Workflow name: Result monitoring			Project Manager
 Stadiert_based Gradiert_based Gradiert_based Gradiert_based Stadiert_based Stadiert_based Stadiert_based Meta_modeling_META Adaptive_response_surface_ARSM Robustness_analysis Robustness_analysis Robust_design_optimization_RDO Robust_design_optimization_RDO Revaluate_results Result_monitoring 	Description: Open and show a result file (resp. Save_xyz.b) Workflow Parameters Mode and Files Mode Autom	n Select a task Proje Open project New project Delete project	Import Project roject name: tet directory: Project of: Projects Axial MonineFSliblend
Select		Import project Customize project	
.ook <u>I</u> n: optiSLang			
RSM_ARSM bin			
Siblend optiSLang dsa DOE 🛛 🗂 opti proble	ems		Apply Dack Cancel Close
Siblend_optiSLang_rob_ROBUST 🛛 workflows	iSLang\4	xialTurbineARSM_ARSM/Save_AxialTurbineAR_ARSM.bin	
SIblend_optiSLang_robust_ROBUST 🗋 AxialTurbio	neFSIb		
SIblend_optiSLang_SA2_DOE			
ile Name: AxialTurbineFSlblend.fgpr		Start Save Reset Stop	
iles of Type: flowguide project files	▼ ring_par	am_090807_172625 : AxialTurbineFSIblend	
	Select Cancel		
Result:			
optiSI and 3.0.1 powered by <i>flowGuide</i> Started workflow: Pe	sult monitoring instance		

Workbench Interface to optiSLang



© 2010 ANSYS, Inc. All rights reserved.

Process Integration

Sensitivity Analysis

Design Optimization

Robustness Evaluation

Random Fields



eliability Analysis

ANSYS®





© 2010 ANSYS, Inc. All rights reserved.

57



OAnalysis of parameter sensitivity means

- Oinvestigating the effect of variability (sensitivity) of certain input parameters on the variability of designrelevant response quantities,
- Oidentification of the most important input parameters,
- **Oidentification of linear and nonlinear sensitivities**,
- **Oidentification of multivariate dependencies**,
- Obuilding metamodels for reducing CAE or experimental costs and gain of knowledge,
- Quantification of the model or experimental predictability and noise fraction and
- Osearching best input parameters vector as starting point for design optimization.





- investigating the input parameter variability effect on the
- O variability of the responses
- identification of the most important input parameters
- identification of linear and nonlinear sensitivities
- quantification of the model predictability

50 200 INPUT:

ANSYS®

- Latin hypercube sampling to
 - Minimize the correlation error of the input variables and to
 - Simulate uniform distributed uncorrelated design variables
 - Required design evaluations
 N > 50...100





60





adjusted CoI [%] of OUTPUT: norm 2 diff disp

 Linear single parameter correlations

Without linear correlation < 0.3

INPUT: E vs. OUTPUT: norm_2_diff_disp, (linear) r = -0.216 0.45 _2_diff_disp 0.35 2 0UTPUT: norm 0.15 0.25 05 9.5 1.75 2.25 2.5 2.75 3 3.25 3.5 2 INPUT: E [1e10]



INPUT: damp1 vs. OUTPUT: norm_2_diff_disp, (linear) r = -0.406



Strong linear correlation > 0.7

ANSYS[®]







© 2010 ANSYS, Inc. All rights reserved.

ANSYS[®]

Nonlinear monotonic single \bigcirc parameter correlations (Spearman correlation)



Without linear correlation < 0.3



Strong linear correlation > 0.7



64

0.045

Weak linear correlation < 0.5

ANSYS, Inc. Proprietary





ANSYS[®] dynancio

- General nonlinear multivariate parameter correlations (CoP)
- Visualization of the physical parameter dependencies and relationships
- Metamodel of optimal prognosis can be used for optimization and stochastic analysis



Small influence CoP <10%



Weak influence CoP <15%

Strong influence CoP >15%







Correlation Matrix







© 2010 ANSYS, Inc. All rights reserved.

ANSYS[®]

71





© 2010 ANSYS, Inc. All rights reserved.

ANSYS, Inc. Proprietary





© 2010 ANSYS, Inc. All rights reserved.

ANSYS, Inc. Proprietary








Visualization of MoP





Parameter Reduction



Important Parameter:

- InletWidth
- ExitWidth
- RImpeller

-46

-48

-50

10

INPUT: ShdBeta -58 -56 -54 -5

<u>9</u>

-62

64

0.8

RV/Hub/Shd Beta 1,3



Conclusion Sensitivity Analysis

- Statistic is reliable, QoP>80%
- Parameter Reduction possible
- Statistic Shows Optimization Potential



1.28 1.32 1.36 1.3 1.34 OUTPUT: VTF pr tt

-60

-55

-50

ANSYS[®]

0.825 0.824 0.823 0.822

0.821 0.820 0.819 0.818

0.817 0.816 0.815 0.814 0.813 0.812

0.811 0.810

0.809

0.808

MLS approximation of VTF etas tt Coefficient of Prognosis =

89 %

© 2010 ANSYS, Inc. All rights reserved.

0.805

Conclusion Sensitivity Analysis

OEfficiency: myeta

- InletWidth : as smallest as possible
- ShdBeta1 : as smallest as possible
- O HubBeta1 : as smallest as possible

O RImpeller : as smallest as possible

 RVShdBeta1 : as largest as possible

• RVHubBeta1 : as largest as possible

Opti	Robust O	utput String	s Constrain	ts Objective	s			
#	Name	Value	Ref.Value	Lower Bound	Upper Bound	Туре	Format	Active Con
1	InletWidth	53	49.237	53.5	57.5	continuous	%20.14f	
2	ExitWidth	26	26.91	26.5	28.5	continuous	%20.14f	
3	Rimpeller	305	288.3	292.0	298.0	continuous	%20.14f	
4	HubBeta1	-48	-51.888	-52.5	-49.5	continuous	%20.14f	
5	HubBeta3	-25	-25.075	-27.5	-26.5	continuous	%20.14f	
6	ShdBeta1	-55	-58.355	-60.5	-59.5	continuous	%20.14f	
7	RVHubBeta1	60	64.02	62.0	66.0	continuous	%20.14f	
8	RVShdBeta1	60	69.5	60.0	64.0	continuous	%20.14f	
9	mymassin	72.6	72.6	65.34	87.119999999	continuous	%20.14f	
10	myomega	699.76	699.76	629.784	839.712	continuous	%20.14f	
11	HubBeta2	-25	-26.525	-27.5	-22.5	continuous	%20.14f	
12	ShdBeta2	-45	-42.885	-49.5	-40.5	continuous	%20.14f	
13	ShdBeta3	-30	-30.69	-33.0	-27.0	continuous	%20.14f	
14	HubThk1	1	0.818	0.8	1.2	continuous	%20.14f	
15	HubThk2	6	6.01	5.0	7.0	continuous	%20.14f	
16	ShdThk1	1	0.93	0.8	1.2	continuous	%20.14f	
17	ShdThk2	6	6.85	5.0	7.0	continuous	%20.14f	
18	RVHubThk1	45	61.065	45.0	66.0	continuous	%20.14f	
19	RVShdThk1	45	38.3	35.0	55.0	continuous	%20.14f	
20	ImpellerBlades	20	20	18.0	24.0	continuous	%20.14f	
21	RVBlades	24	24	21.6	28.799999999	continuous	%20.14f	

Cancel

OK

OTotal pressure: ptratio

- O ShdBeta1 : as smallest as possible
- RImpeller : as largest as possible
- HubBeta3 : as smallest as possible
- O ExitWidth : as largest as possible
- O InletWidth : as smallest as possible

Process Integration

Sensitivity Analysis

Design Optimization Robustness Evaluation

ANSYS® dynando

1



Random Fields

eliability Analysis

Design Optimization





Optimization Strategy



Sensitivity Analysis (SA)

- Shows optimization potential
- Indicates start solution
- Parameter reduction
- Modify parameter space



Strategy:

- Start design(s) from SA
- Pre-optimization in sub space, ARSM
- Start design(s) from ARSM
- Local improvement, EA (full space)



Adaptive Response Surface Methods (Local)





Adaptive Response Surface

ARSM with 8 performance-relevant

parameters

Opti	Robust 0	utput String	s Constrain	ts Objective	s			
#	Name	Value	Ref.Value	Lower Bound	Upper Bound	Туре	Format	Active Con
1	InletWidth	53	49.237	53.5	57.5	continuous	%20.14f	
2	ExitWidth	26	26.91	26.5	28.5	continuous	%20.14f	
3	Rimpeller	305	288.3	292.0	298.0	continuous	%20.14f	
4	HubBeta1	-48	-51.888	-52.5	-49.5	continuous	%20.14f	
5	HubBeta3	-25	-25.075	-27.5	-26.5	continuous	%20.14f	
6	ShdBeta1	-55	-58.355	-60.5	-59.5	continuous	%20.14f	
7	RVHubBeta1	60	64.02	62.0	66.0	continuous	%20.14f	
8	RVShdBeta1	60	69.5	60.0	64.0	continuous	%20.14f	
9	mymassin	72.6	72.6	65.34	87.119999999	continuous	%20.14f	
10	myomega	699.76	699.76	629.784	839.712	continuous	%20.14f	
11	HubBeta2	-25	-26.525	-27.5	-22.5	continuous	%20.14f	
12	ShdBeta2	-45	-42.885	-49.5	-40.5	continuous	%20.14f	
13	ShdBeta3	-30	-30.69	-33.0	-27.0	continuous	%20.14f	
14	HubThk1	1	0.818	0.8	1.2	continuous	%20.14f	
15	HubThk2	6	6.01	5.0	7.0	continuous	%20.14f	
16	ShdThk1	1	0.93	0.8	1.2	continuous	%20.14f	
17	ShdThk2	6	6.85	5.0	7.0	continuous	%20.14f	
18	RVHubThk1	45	61.065	45.0	66.0	continuous	%20.14f	
19	RVShdThk1	45	38.3	35.0	55.0	continuous	%20.14f	
20	ImpellerBlades	20	20	18.0	24.0	continuous	%20.14f	
21	RVBlades	24	24	21.6	28.799999999	continuous	%20.14f	

Cancel

ANSYS[®]

OK

Adaptive Response Surface



ARSM leads to better design:



Adaptive Response Surface





Evolutionary Algorithm



EA with 17 parameters and 74 constraints leads to further improvement:

Opti	Robust 0	utput String	s Constraint	s Objectives	;				
#	Name	Value	Ref.Value	Lower Bound	Upper Bound	Туре	Format	Acti	
1	InletWidth	53	53.613661065	53.5	57.5	continuous	%20.14f		
2	ExitWidth	26	27.615811531	26.5	28.5	continuous	%20.14f	~	
3	Rimpeller	305	292.61382989	292.0	298.0	continuous	%20.14f	~	
4	HubBeta1	-48	-52.34845257	-52.5	-49.5	continuous	%20.14f	~	
5	HubBeta2	-25	-26.525	-27.5	-22.5	continuous	%20.14f	~	
6	HubBeta3	-25	-27.01713251	-27.5	-26.5	continuous	%20.14f		
7	ShdBeta1	-55	-60.26762316	-60.5	-59.5	continuous	%20.14f	V	
8	ShdBeta2	-45	-42.885	-49.5	-40.5	continuous	%20.14f	~	
9	ShdBeta3	-30	-30.69	-33.0	-27.0	continuous	%20.14f	V	
10	HubThk1	1	0.818	0.8	1.2	continuous	%20.14f		
11	HubThk2	6	6.01	5.0	7.0	continuous	%20.14f		
12	ShdThk1	1	0.93	0.8	1.2	continuous	%20.14f	V	
13	ShdThk2	6	6.85	5.0	7.0	continuous	%20.14f	~	
14	RVHubThk1	45	61.065	45.0	66.0	continuous	%20.14f	~	
15	RVHubBeta1	60	65.526394007	62.0	66.0	continuous	%20.14f	~	
16	RVShdBeta1	60	63.123088351	60.0	64.0	continuous	%20.14f		
17	RVShdThk1	45	38.3	35.0	55.0	continuous	%20.14f	~	
18	mymassin	72.6	72.6	65.34	87.119999999	continuous	%20.14f		
19	myomega	699.76	699.76	629.784	839.712	continuous	%20.14f		
20	ImpellerBlades	20	20	18.0	24.0	continuous	%20.14f		
21	RVBlades	24	24	21.6	28.799999999	continuous	%20.14f		

# Name Type Formula Adive 14 relTmodeh021 ne fabs.(6.8232*modeTreq022-(2*myomega)/(2*myomega)).0.01 IZ 15 relTmodeh022 ne fabs.(6.8232*modeTreq023-(2*myomega)/(2*myomega)).0.01 IZ 16 relZmodeh022 ne fabs.(6.8232*modeTreq031-(2*myomega)/(2*myomega)).0.01 IZ 19 relTmodeh031 ne fabs.(6.8232*modeTreq031-(2*myomega)/(2*myomega)).0.01 IZ 21 relZmodeh031 ne fabs.(6.8232*modeTreq031-(2*myomega)/(2*myomega)).0.01 IZ 22 relZmodeh032 ne fabs.(6.8232*modeTreq031-(2*myomega)/(2*myomega)).0.01 IZ 23 relTmodeh033 ne fabs.(6.8232*modeTreq031-(2*myomega)/(2*myomega)).0.01 IZ 24 relZmodeh043 ne fabs.(6.8232*modeTreq031-(1*myomega)/(1*myomega)).0.01 IZ 25 relTmodeh044 ne fabs.(6.8232*modeTreq031-(1*myomega)/(1*myomega)).0.01 IZ 26 relTmodeh044 ne fabs.(6.8232*modeTreq031-(1*myomega)/(1*myomega)).0.01 IZ 27 relTmodeh045 ne.	Opti	Robust 0	utput	Strings	Constraints	Objectives		
14 rel2modeh021 ine fabs(6 2832*modefreq022-1(**mymega))/(2*mymega))-0.11 V 15 rel1modeh022 ine fabs(6 2832*modefreq022-2(**mymega))/(2*mymega))-0.01 V 16 rel2modeh023 ine fabs(6 2832*modefreq023-2(**mymega))/(2*mymega))-0.01 V 17 rel1modeh023 ine fabs(6 2832*modefreq023-2(**mymega))/(2*mymega))-0.01 V 19 rel1modeh031 ine fabs(6 2832*modefreq023-2(**mymega))/(2*mymega))-0.01 V 21 rel2modeh032 ine fabs(6 2832*modefreq033-2(**mymega))/(2*mymega))-0.01 V 22 rel2modeh032 ine fabs(6 2832*modefreq033-1(**mymega))/(1*mymega))-0.01 V 23 rel1modeh033 ine fabs(6 2832*modefreq043-1(**mymega))/(1*mymega))-0.01 V 24 rel2modeh043 ine fabs(6 2832*modefreq042-1(**mymega))/(1*mymega))-0.01 V 25 rel1modeh044 ine fabs(6 2832*modefreq042-1(**mymega))/(1*mymega))-0.01 V 26 rel2modeh044 ine fabs(6 2832*modefreq042-1(**mymega))/(2*mymega))-0.01 V 27	#	Name	Туре			ormula		Active
15 relTmodeh022 ine fabs/(6.2832*modefreq022-2(*mymonga))/(*mymonga))-0.01 IV 16 relZmodeh022 ine fabs/(6.2832*modefreq023-2(*mymonga))/(2*mymonga))-0.01 IV 17 relTmodeh023 ine fabs/(6.2832*modefreq023-1(*mymonga))/(2*mymonga))-0.01 IV 18 relZmodeh033 ine fabs/(6.2832*modefreq031-1(*mymonga))/(2*mymonga))-0.01 IV 20 relZmodeh033 ine fabs/(6.2832*modefreq032-1(*mymonga))/(2*mymonga))-0.01 IV 21 relTmodeh033 ine fabs/(6.2832*modefreq031-12*mymonga))/(1*mymonga))-0.01 IV 23 relZmodeh041 ine fabs/(6.2832*modefreq031-2*mymonga))/(2*mymonga))-0.01 IV 24 relZmodeh041 ine fabs/(6.2832*modefreq041-2*mymonga))/(1*mymonga))-0.01 IV 25 relZmodeh041 ine fabs/(6.2832*modefreq041-2*mymonga)/(1*mymonga))-0.01 IV 26 relZmodeh042 ine fabs/(6.2832*modefreq041-2*mymonga)/(1*mymonga))-0.01 IV 27 relTmodeh043 ine fabs/(6.2832*modefreq031-1*mymonga)/(1*mymonga))-0.01 IV <td< td=""><td>14</td><td>rel2modeh021</td><td>ine</td><td>fabs((6.2</td><td>32*modefreq021-</td><td>(2*myomega))/(</td><td>2*myomega))-0.01</td><td>2</td></td<>	14	rel2modeh021	ine	fabs((6.2	32*modefreq021-	(2*myomega))/(2*myomega))-0.01	2
16 rel/modeh023 ne fabs(6.2322/modefreq023-(1*myomega))/(1*myomega)-0.01 V 17 rel/modeh023 ne fabs(6.2322/modefreq023-(1*myomega))/(1*myomega)-0.01 V 19 rel/modeh031 ne fabs(6.2322/modefreq032-(2*myomega))/(2*myomega))-0.01 V 19 rel/modeh031 ne fabs(6.2322/modefreq032-(2*myomega))/(2*myomega))-0.01 V 21 rel/modeh032 ne fabs(6.2332/modefreq032-(1*myomega))/(1*myomega))-0.01 V 22 rel/modeh031 ne fabs(6.2332/modefreq032-(1*myomega))/(2*myomega))-0.01 V 23 rel/modeh041 ne fabs(6.2332/modefreq041-(1*myomega))/(2*myomega))-0.01 V 24 rel/modeh042 ne fabs(6.2332/modefreq042-(2*myomega))/(2*myomega))-0.01 V 25 rel/modeh042 ne fabs(6.2332/modefreq042-(2*myomega))/(2*myomega))-0.01 V 26 rel/modeh042 ne fabs(6.2332/modefreq042-(2*myomega))/(2*myomega))-0.01 V 27 rel/modeh043 ne fabs(6.2332/modefreq042-(2*myomega))/(2*myomega))-0.01 V 23	15	rel1modeh022	ine	fabs((6.2	32*modefreq022-	(1*myomega))/(1*myomega))-0.01	V
17 ret/modeh023 ne. fbbs(6/2332/modefreq023-(1/myornega))/(1/myornega)-0.01 V 18 ret/modeh031 ne. fbbs(6/2332/modefreq023-(1/myornega))/(2/myornega)-0.01 V 19 ret/modeh031 ne. fbbs(6/2332/modefreq03-(1/myornega))/(2/myornega))-0.01 V 20 ret/modeh032 ne. fbbs(6/2332/modefreq03-(2/myornega))/(2/myornega))-0.01 V 21 ret/modeh033 ne. fbbs(6/2332/modefreq03-(2/myornega))/(2/myornega))-0.01 V 22 ret/modeh041 ne. fbbs(6/2332/modefreq04-1/2/myornega)/(2/myornega))-0.01 V 23 ret/modeh042 ne. fbbs(6/2332/modefreq04-2/myornega)/(2/myornega))-0.01 V 24 ret/modeh042 ne. fbbs(6/2332/modefreq04-2/myornega)/(1/myornega))-0.01 V 25 ret/modeh042 ne. fbbs(6/2332/modefreq04-2/myornega)/(1/myornega))-0.01 V 26 ret/modeh043 ne. fbbs(6/2332/modefreq04-2/myornega)/(1/myornega))-0.01 V 27 ret/modeh051 ne. fbbs(6/2332/modefreq04-2/myornega)/(1/myornega)/0.01 V 31 ret/modeh051	16	rel2modeh022	ine	fabs((6.2	32*modefreq022-	(2*myomega))/(2*myomega))-0.01	V
16 rel/modeh031 ne. fabs(6.2332/modefreq031-(1myomega))/(1myomega)-0.01 V 19 rel/modeh031 ne. fabs(6.2332/modefreq031-(1myomega))/(1myomega)-0.01 V 21 rel/modeh032 ne. fabs(6.2332/modefreq032-(1myomega))/(1myomega)-0.01 V 22 rel/modeh032 ne. fabs(6.2332/modefreq032-(1myomega))/(1myomega)-0.01 V 23 rel/modeh033 ne. fabs(6.2332/modefreq032-(1myomega))/(1myomega)-0.01 V 24 rel/modeh031 ne. fabs(6.2332/modefreq042-(1myomega))/(2myomega)-0.01 V 25 rel/modeh041 ne. fabs(6.2332/modefreq042-(1myomega))/(2myomega)-0.01 V 26 rel/modeh042 ne. fabs(6.2332/modefreq042-(1myomega))/(2myomega)-0.01 V 27 rel/modeh043 ne. fabs(6.2332/modefreq042-(1myomega))/(2myomega)-0.01 V 28 rel/modeh043 ne. fabs(6.2332/modefreq042-(1myomega))/(2myomega)-0.01 V 30 rel/modeh051 ne. fabs(6.2332/modefreq052-(1myomega))/(2myomega)-0.01 V 31 rel/modeh051 ne.	17	rel1modeh023	ine	fabs((6.2	332*modefreq023-	-(1*myomega))/(1*myomega))-0.01	V
19 relfmodeh031 ne fabs(6.2832*modefreq031-(1*myomega))/(1*myomega)-0.01 If 20 re2modeh032 ine fabs(6.2832*modefreq032-(2*myomega))/(2*myomega)-0.01 If 21 re1modeh032 ine fabs(6.2832*modefreq032-(2*myomega))/(2*myomega))-0.01 If 22 re1modeh033 ine fabs(6.2832*modefreq032-(2*myomega))/(2*myomega))-0.01 If 23 re1modeh031 ine fabs(6.2832*modefreq042-(1*myomega))/(2*myomega))-0.01 If 24 re2modeh041 ine fabs(6.2832*modefreq042-(1*myomega))/(1*myomega))-0.01 If 25 re1modeh042 ine. fabs(6.2832*modefreq042-(1*myomega))/(1*myomega))-0.01 If 26 re2modeh042 ine. fabs(6.2832*modefreq042-(1*myomega))/(1*myomega))-0.01 If 27 re1modeh043 ine. fabs(6.2832*modefreq051-(2*myomega))/(1*myomega))-0.01 If 28 re2modeh051 ine. fabs(6.2832*modefreq052-(1*myomega))/(2*myomega))-0.01 If 29 re1modeh052 ine. fabs(6.2832*modefreq052-(1*myomega))/(2*myomega))-0.01 If 29 <	18	rel2modeh023	ine	fabs((6.2	332*modefreq023-	-(2*mvomega))/(2*mvomega))-0.01	V
20 relZmodeh031 le fbbs(6.2832*modefreq03-1(2*myonega))(2*myonega))-0.01 IV 21 relZmodeh032 le fbbs(6.2832*modefreq032-(1*myonega))(2*myonega))-0.01 IV 23 relTmodeh033 le fbbs(6.2832*modefreq032-(1*myonega))(2*myonega))-0.01 V 24 relZmodeh033 le fbbs(6.2832*modefreq032-(1*myonega))(2*myonega))-0.01 V 25 relTmodeh031 le fbbs(6.2832*modefreq042-(1*myonega))(2*myonega))-0.01 V 26 relZmodeh042 le fbbs(6.2832*modefreq042-(1*myonega))(2*myonega))-0.01 V 27 relTmodeh043 le fbbs(6.2832*modefreq042-(1*myonega))(2*myonega))-0.01 V 28 relZmodeh043 le fbbs(6.2832*modefreq051-(2*myonega))(2*myonega))-0.01 V 30 relZmodeh051 le fbbs(6.2832*modefreq052-(1*myonega))(2*myonega))-0.01 V 31 relTmodeh051 le fbbs(6.2832*modefreq051-(2*myonega))(2*myonega))-0.01 V 33 relTmodeh051 le	19	rel1modeh031	ine	fabs((6.2	332*modefreq031-	-(1*mvomega))/(1*myomega))-0.01	V
21 refIrmodeh032 ne. fabs(6: 2832*modefreq032.(1*myomega))(2*myomega)-0.01 V/ 22 resImodeh033 ne. fabs(6: 2832*modefreq033.(2*myomega)).0.01 V/ 23 resImodeh033 ne. fabs(6: 2832*modefreq033.(2*myomega)).0.01 V/ 24 resImodeh031 ne. fabs(6: 2832*modefreq041.(*myomega)).0.01 V/ 25 resImodeh041 ne. fabs(6: 2832*modefreq042.(2*myomega)).0.01 V/ 25 resImodeh042 ne. fabs(6: 2832*modefreq042.(2*myomega)).(2*myomega)).0.01 V/ 26 resImodeh043 ne. fabs(6: 2832*modefreq042.(2*myomega)).(1*myomega)).0.01 V/ 27 resImodeh043 ne. fabs(6: 2832*modefreq051.(*myomega)).0.01 V/ 31 resImodeh051 ne. fabs(6: 2832*modefreq052.(*myomega)).0.01 V/ 33 resImodeh053 ne. fabs(6: 2832*modefreq053.(*myomega)).0.01 V/ 33 resImodeh053 ne. fabs(6: 2832*modefreq053.(*myomega)).0.01 V/ 34 resImodeh053 ne. fabs(6: 2832*modefreq053.(*myomega)).0.01	20	rel2modeh031	ine	fabs((6.2	332*modefreq031-	-(2*myomega))/(2*myomega))-0.01	V
22 rel2modeh032 ine fabs((6.2832*modefreq03.2(*myomega))/(2*myomega)).0.01 V 23 rel1modeh033 ine fabs((6.232*modefreq03.2(*myomega)).0.01 V 24 rel2modeh033 ine fabs((6.232*modefreq03.2(*myomega)).0.01 V 25 rel1modeh041 ine fabs((6.232*modefreq04.2(*myomega)).0.01 V 27 rel1modeh042 ine fabs((6.2332*modefreq04.2(*myomega)).0.01 V 28 rel2modeh043 ine fabs((6.2332*modefreq04.3(*myomega)).0.01 V 29 rel1modeh043 ine fabs((6.2332*modefreq05.4(*myomega)).0(?myomega)).0.01 V 30 rel2modeh052 ine fabs((6.2332*modefreq05.4(*myomega)).0(?myomega)).0.01 V 31 rel1modeh052 ine fabs((6.2332*modefreq05.2(*myomega)).0(?myomega)).0.01 V 33 rel1modeh052 ine fabs((6.2332*modefreq05.2(*myomega)).0.1(*myomega)).0.01 V 34 rel2modeh052 ine fabs((6.2332*modefreq05.2(*myomega)).0.21* V 35 rel1modeh052 ine fab	21	rel1modeh032	ine	fabs((6.2	332*modefreq032	-(1*mvomega))/(1*mvomega))-0.01	r r
23 relimodeh033 ine fabs((6.2832*modefreq03.12*myomega))(2*myomega).0.01 V 24 relimodeh041 ine fabs((6.2832*modefreq04.12*myomega))(2*myomega).0.01 V 25 relimodeh041 ine fabs((6.2832*modefreq04.12*myomega))(2*myomega).0.01 V 26 relimodeh042 ine fabs((6.2832*modefreq04.21*myomega))(2*myomega).0.01 V 27 relimodeh043 ine fabs((6.2832*modefreq04.22*myomega))(2*myomega).0.01 V 28 relimodeh043 ine fabs((6.2832*modefreq04.22*myomega))(2*myomega).0.01 V 29 relimodeh043 ine fabs((6.2832*modefreq05.14*myomega))(1*myomega).0.01 V 31 relimodeh051 ine fabs((6.2832*modefreq05.24*myomega))(2*myomega).0.01 V 33 relimodeh051 ine fabs((6.2832*modefreq05.1*myomega))(1*myomega).0.01 V 34 relimodeh051 ine fabs((6.2832*modefreq05.1*myomega))(2*myomega).0.01 V 35 relimodeh061 ine fabs((6.2832*modefreq05.1*myomega))(2*myomega).0.01 V 36 reli	22	rel2modeh032	ine	fabs((6.2	332*modefreq032	-(2*mvomega))/(2*mvomega))-0.01	r r
24 rel2modeh033 im. fabs((6.2832*modefreq03.3)(2*myomega))(2*myomega)).0.01 Im. 25 rel7modeh041 ine fabs((6.2832*modefreq04.1(*myomega))(2*myomega)).0.01 Im. 26 rel7modeh041 ine fabs((6.2832*modefreq04.2(*myomega))(2*myomega)).0.01 Im. 27 rel7modeh043 ine fabs((6.2832*modefreq04.3(*myomega))(2*myomega)).0.01 Im. 28 rel2modeh043 ine fabs((6.2832*modefreq04.3(*myomega))(2*myomega)).0.01 Im. 29 rel1modeh051 ine fabs((6.2832*modefreq05.1(*myomega))(2*myomega)).0.01 Im. 30 rel2modeh051 ine fabs((6.2832*modefreq05.2(*myomega))(2*myomega)).0.01 Im. 33 rel1modeh051 ine fabs((6.2832*modefreq05.2(*myomega))(2*myomega)).0.01 Im. 34 rel2modeh051 ine fabs((6.2832*modefreq05.2(*myomega))(1*myomega)).0.01 Im. 35 rel1modeh051 ine fabs((6.2832*modefreq05.2(*myomega))(1*myomega)).0.01 Im. 36 rel2modeh052 ine fabs((6.2832*modefreq05.2(*myomega))(1*myomega)).0.01 Im.	23	rel1modeh033	ine	fabs((6.2	32*modefreq033	(1*mvomega))/(1*myomega))-0.01	
25 rel1modeh041 nm fabs((6.2332*modefreq041-(1*myomega))/(2*myomega))-0.01 V 28 re2modeh042 nm fabs((6.2332*modefreq042-(2*myomega))/(2*myomega))-0.01 V 28 re2modeh042 nm fabs((6.2332*modefreq043-(2*myomega))/(2*myomega))-0.01 V 29 re1modeh043 nm fabs((6.2332*modefreq043-(2*myomega))/(2*myomega))-0.01 V 30 re2modeh043 nm fabs((6.2332*modefreq053-(1*myomega))/(2*myomega))-0.01 V 31 re1modeh051 nm fabs((6.2332*modefreq053-(1*myomega))/(2*myomega))-0.01 V 33 re1modeh053 nm fabs((6.2332*modefreq053-(1*myomega))/(2*myomega))-0.01 V 34 re2modeh053 nm fabs((6.2332*modefreq053-(1*myomega))/(2*myomega))-0.01 V 35 re1modeh051 nm fabs((6.2332*modefreq053-(1*myomega))/(2*myomega))-0.01 V 36 re2modeh062 nm fabs((6.2332*modefreq053-(1*myomega))/(2*myomega))-0.01 V 37 re1modeh061 nm fabs((6.2332*modefreq052-(2*myomega))/(2*myomega))-0.01 V 38	24	rel2modeh033	ine	fabs((6.2	32*modefreq033-	(2*myomega))/(2*myomega))-0.01	
10.1mden0141 Inc. ftp3(12202 22 rel2modeh041 Inc. ftp3(12202 23 rel2modeh042 Inc. ftp3(12202 24 rel2modeh042 Inc. ftp3(12202 25 rel2modeh043 Inc. ftp3(12202 30 rel2modeh043 Inc. ftp3(12202 31 rel1modeh051 Inc. ftp3(12202 31 rel1modeh051 Inc. ftp3(12202 33 rel1modeh052 Inc. ftp3(12202 34 rel2modeh051 Inc. ftp3(12202 35 rel1modeh052 Inc. ftp3(12202 36 rel2modeh051 Inc. ftp3(12202 37 rel1modeh051 Inc. ftp3(12202 38 rel2modeh051 Inc. ftp3(12202 39 rel1modeh052 Inc. ftp3(12202 39 rel1modeh052 Inc. ftp3(12202 39 rel1modeh052 Inc. ftp3(12202 39	25	rel1modeh041	ine	fabs((6.2	832*modefreq041.	(1*myomena))/(1*myomena))_0.01	
2: reitimodeh02 inc ftabs((2.232*modefreq042-2(1*myomega))/(2*myomega)).0.01 P 28 rel2modeh042 inc ftabs((6.2332*modefreq042-2(1*myomega))/(2*myomega)).0.01 P 30 rel2modeh043 inc ftabs((6.2332*modefreq043-2(1*myomega))/(2*myomega)).0.01 P 31 rel1modeh051 inc ftabs((6.2332*modefreq051-12*myomega))/(2*myomega)).0.01 P 32 rel2modeh051 inc ftabs((6.2332*modefreq051-2(2*myomega))/(2*myomega)).0.01 P 33 rel1modeh052 inc ftabs((6.2332*modefreq051-2(*myomega))/(2*myomega)).0.01 P 34 rel2modeh051 inc ftabs((6.2332*modefreq051-2(*myomega))/(2*myomega)).0.01 P 35 rel1modeh061 inc ftabs((6.2332*modefreq051-(1*myomega))/(2*myomega)).0.01 P 36 rel2modeh061 inc ftabs((6.2332*modefreq051-(2*myomega))/(2*myomega)).0.01 P 37 rel1modeh062 inc ftabs((6.2332*modefreq051-(2*myomega))/(2*myomega)).0.01 P 38 rel2modeh062 inc ftabs((6.2332*modefreq071-(2*myomega))/(2*myomega)).0.01 P <	26	rel2modeh041	ine	fabs((6.2	832*modefreq041	(2*myomega))/(2*myomega))_0.01	
1 101m0den/21 inc ftabs((6.2832*modefreq042-(2*myomega))(2*myomega))-0.01 2 28 reiEmodeh042 inc ftabs((6.2832*modefreq042-(2*myomega))(2*myomega))-0.01 2 30 reiEmodeh043 inc ftabs((6.2832*modefreq051-(1*myomega))(2*myomega))-0.01 2 31 reiEmodeh051 inc ftabs((6.2832*modefreq052-(1*myomega))(2*myomega))-0.01 2 32 reiEmodeh052 inc ftabs((6.2832*modefreq053-(2*myomega))(2*myomega))-0.01 2 34 reiEmodeh053 inc ftabs((6.2832*modefreq053-(2*myomega))(2*myomega))-0.01 2 35 reiImodeh061 inc ftabs((6.2832*modefreq053-(2*myomega))(2*myomega))-0.01 2 36 reiEmodeh061 inc ftabs((6.2832*modefreq053-(2*myomega))(2*myomega))-0.01 2 37 reiImodeh062 inc ftabs((6.2832*modefreq053-(2*myomega))(2*myomega))-0.01 2 38 reiEmodeh061 inc ftabs((6.2832*modefreq071-(1*myomega))(1*myomega))-0.01 2 40 reiEmodeh062 inc ftabs((6.2832*modefreq071-(2*myomega))(2*myomega))-0.01 2	27	rel1modeh042	ine	fabe((6.2	32*modefreq042	(1*myomega))/(1*myomega)) 0.01	
102.102.002.102.002 112.102 22 reitmodeh033 in fabs((6.2832*modefreq043.2(*myomega))(2*myomega)).0.01 12 33 reitmodeh031 in fabs((6.2832*modefreq051.1(*myomega))(2*myomega)).0.01 12 34 reitmodeh051 in fabs((6.2832*modefreq052.1(*myomega))(2*myomega)).0.01 12 35 reitmodeh052 in fabs((6.2832*modefreq052.2*myomega))(2*myomega)).0.01 12 36 reitmodeh053 in fabs((6.2832*modefreq052.2*myomega))(2*myomega)).0.01 12 36 reitmodeh053 in fabs((6.2832*modefreq052.2*myomega))(2*myomega)).0.01 12 37 reitmodeh051 in fabs((6.2832*modefreq052.1*myomega))(2*myomega)).0.01 12 38 reitmodeh052 in fabs((6.2832*modefreq053.2*myomega))(2*myomega)).0.01 12 40 reitmodeh052 in fabs((6.2832*modefreq053.2*myomega))(2*myomega)).0.01 12 41 reitmodeh052 in fabs((6.2832*modefreq053.2*myomega))(2*myomega)).0.01 12 42 reitmodeh053 in fabs((6.2832*modefreq053.2*myomeg	28	rel2modeb042	ine	fabe((6.2	332*modefreq042	(2*myomega))/(2*myomega)) 0.01	
25 Tell modeln03 Inc Tabs(IC 2332*modefreq03-24; 2*myomega)/(2*myomega):0.01 Image/10.01 31 relEmodeln03 inc fabs(IC 2332*modefreq05-1(2*myomega))/(2*myomega):0.01 Image/10.01 32 reZmodeln051 inc fabs(IC 2332*modefreq05-1(2*myomega))/(2*myomega):0.01 Image/10.01 33 reImodeln051 inc fabs(IC 2332*modefreq05-2(1*myomega))/(2*myomega):0.01 Image/10.01 34 reImodeln051 inc fabs(IC 2332*modefreq05-3(1*myomega))/(2*myomega):0.01 Image/10.01 35 reImodeln051 inc fabs(IC 2332*modefreq05-2(*myomega))/(2*myomega):0.01 Image/10.01 36 reImodeln061 inc fabs(IC 2332*modefreq05-2(*myomega))/(2*myomega):0.01 Image/10.01 37 reImodeln062 inc fabs(IC 2332*modefreq05-2(*myomega))/(2*myomega):0.01 Image/10.01 38 reImodeln051 inc fabs(IC 2332*modefreq05-2(*myomega))/(2*myomega):0.01 Image/10.01 39 reImodeln062 inc fabs(IC 2332*modefreq07-1(1*myomega))/(1*myomega):0.01 Image/10.01 Image/10.01 Image/10.01 Image/10.01 Image/10.01	20	rol1modoh042	ino	fabs((6.2	222tmodofrog042	(1*mvomoga))/(1*mvomoga)) 0.01	-
30 reitmodeh051 inc fabsi(0.2322*modefreq051-1(*myomega))(2*myomega)-0.01 Ľ 32 reitmodeh051 inc fabsi(0.2322*modefreq051-1(*myomega))(2*myomega)-0.01 Ľ 33 reitmodeh052 inc fabsi(0.232*modefreq052-1(*myomega))(2*myomega)-0.01 Ľ 34 reiZmodeh052 inc fabsi(0.232*modefreq052-1(*myomega))(2*myomega)-0.01 Ľ 35 reitmodeh053 inc fabsi(0.232*modefreq052-12*myomega))(1*myomega)-0.01 Ľ 36 reiZmodeh053 inc fabsi(0.233*modefreq052-14*myomega))(1*myomega)-0.01 Ľ 37 reitmodeh052 inc fabsi(0.233*modefreq052-14*myomega))(1*myomega)-0.01 Ľ 38 reiZmodeh062 inc fabsi(0.233*modefreq052-14*myomega))(1*myomega)-0.01 Ľ 40 reiZmodeh063 inc fabsi(0.233*modefreq052-14*myomega))(1*myomega)-0.01 Ľ 42 reiZmodeh063 inc fabsi(0.233*modefreq072-14*myomega))(1*myomega)-0.01 Ľ 43 reiTmodeh071 inc fabsi(0.233*modefreq073-14*myomega))(1*myomega)-0.01 Ľ 44 <td< td=""><td>20</td><td>rel2modeb043</td><td>inc</td><td>fabs((0.2)</td><td>222modefreq043</td><td>(1 myomega))/(</td><td>2*mvomoga)) 0.01</td><td></td></td<>	20	rel2modeb043	inc	fabs((0.2)	222modefreq043	(1 myomega))/(2*mvomoga)) 0.01	
S1 Tell mödel/031 Tell S2 re2/modeh051 inc fabsi(0.2322*modefreq052.12*myonega)/(2*myonega).0.01 Ľ 33 re1/modeh052 inc fabsi(0.2332*modefreq052.2*myonega)/(2*myonega).0.01 Ľ 34 re2/modeh053 inc fabsi(0.2332*modefreq053.2*myonega)/(2*myonega).0.01 Ľ 35 re1/modeh053 inc fabsi(0.2332*modefreq051.2*myonega)/(2*myonega).0.01 Ľ 36 re1/modeh061 inc fabsi(0.2332*modefreq061.2*myonega)/(2*myonega).0.01 Ľ 37 re1/modeh062 inc fabsi(0.2332*modefreq062.1*myonega)/(2*myonega).0.01 Ľ 38 re1/modeh062 inc fabsi(0.2332*modefreq062.2*myonega)/(2*myonega).0.01 Ľ 44 re2/modeh063 inc fabsi(0.2332*modefreq072.2*myonega)/(2*myonega).0.01 Ľ 44 re2/modeh063 inc fabsi(0.2332*modefreq072.2*myonega)/(2*myonega).0.01 Ľ 44 re2/modeh071 inc fabsi(0.2332*modefreq072.2*myonega)/(2*myonega).0.01 Ľ 44 re2/modeh072 inc fabsi(0.2332*modefreq072.4*myonega	24	rei2modeh043	inc	fabs((0.2	332 modelleq043	-(2 myomega))/(2 myomega))-0.01	
32 rel/modeh052 ine fabs((6.2322*modefreq052.(1*myonega))(2*myonega)).0.01 Ľ 33 rel/modeh052 ine fabs((6.2322*modefreq052.(1*myonega))(2*myonega)).0.01 Ľ 34 rel/modeh052 ine fabs((6.2322*modefreq053.(1*myonega))(2*myonega)).0.01 Ľ 35 rel/modeh053 ine fabs((6.2322*modefreq053.(1*myonega))(1*myonega)).0.01 Ľ 36 rel/modeh061 ine fabs((6.2322*modefreq063.(1*myonega))(1*myonega)).0.01 Ľ 37 rel/modeh061 ine fabs((6.2322*modefreq063.(1*myonega))(1*myonega)).0.01 Ľ 40 rel/modeh062 ine fabs((6.232*modefreq063.(1*myonega))(1*myonega)).0.01 Ľ 41 rel/modeh063 ine fabs((6.232*modefreq071.(1*myonega))(1*myonega)).0.01 Ľ 42 rel/modeh071 ine fabs((6.232*modefreq071.(1*myonega))(1*myonega)).0.01 Ľ 43 rel/modeh071 ine fabs((6.232*modefreq071.(1*myonega))(1*myonega)).0.01 Ľ 44 rel/modeh071 ine	20	rel1moden051	ine	fabs((0.2)	532 mode freq 051	-(1-myomega))/(T-myomega))-0.01	<u>v</u>
33 rel:modef02 inc fabs((6.232*modefreq052.2*myonega))(2*myonega).0.01 Ľ 36 rel:modef052 inc fabs((6.232*modefreq053.2*myonega))(2*myonega).0.01 Ľ 36 rel:modef052 inc fabs((6.232*modefreq053.2*myonega))(2*myonega).0.01 Ľ 37 rel:modef061 inc fabs((6.232*modefreq051.2*myonega))(2*myonega).0.01 Ľ 38 rel:modef062 inc fabs((6.232*modefreq062.1*myonega))(2*myonega).0.01 Ľ 40 rel:modef062 inc fabs((6.232*modefreq063.2*myonega))(2*myonega).0.01 Ľ 41 rel:modef063 inc fabs((6.232*modefreq063.2*myonega))(2*myonega).0.01 Ľ 42 rel:modef071 inc fabs((6.232*modefreq073.1*myonega))(2*myonega).0.01 Ľ 43 rel:modef071 inc fabs((6.232*modefreq073.1*myonega))(2*myonega).0.01 Ľ 44 rel:modef071 inc fabs((6.232*modefreq073.1*myonega))(2*myonega).0.01 Ľ 45 rel:modef071 inc fabs((6.232*modefreq073.2*myonega))(2*myonega).0.01 Ľ 46 rel:modef073<	32	rei2moden051	me	Tabs((0.2)	552 mode freque i	-(2-myomega))/(2-myomega))-0.01	
3-3 relamoden02: nm. Table(0:232*modereq03:1(mmyoneg))(2:myoneg)):0.01 P 36 relamodeh053 inc fabs(0:232*modereq03:2(tmyonega))(2:myonega)):0.01 P 37 relamodeh051 inc fabs(0:232*modereq03:2(tmyonega))(2:myonega)):0.01 P 38 relamodeh061 inc fabs(0:232*modereq06:2(tmyonega))(2:myonega)):0.01 P 39 relamodeh062 inc fabs(0:232*modereq06:2(tmyonega))(1:myonega)):0.01 P 40 relamodeh062 inc fabs(0:232*modereq06:2(tmyonega))(1:myonega)):0.01 P 41 relamodeh063 inc fabs(0:232*modereq07:2(tmyonega))(1:myonega)):0.01 P 42 relamodeh071 inc fabs(0:232*modereq07:1(tmyonega))(1:myonega)):0.01 P 44 relamodeh071 inc fabs(0:232*modereq07:1(tmyonega))(1:myonega)):0.01 P 45 relamodeh072 inc fabs(0:232*modereq07:1(tmyonega))(1:myonega)):0.01 P 46 relamodeh073 inc fabs(0:232*modereq07:1(tmyonega))(1:myonega)):0.01 P 47 relamodeh073	33	rel1moden052	ine	Tabs((6.2	532-modefreqU52-	-(1^myomega))/(1*myomega))-0.01	
35 relmoden053 inc fab8(ic.232*modefreq05.2(*myoneg))/(2*myoneg))-0.01 P 37 relmoden055 inc fab8(ic.232*modefreq05.2(*myoneg))/(2*myoneg))-0.01 P 38 relmoden055 inc fab8(ic.233*modefreq05.2(*myonega))/(2*myonega))-0.01 P 38 relmoden062 inc fab8(ic.233*modefreq05.2(*myonega))/(2*myonega))-0.01 P 40 relmoden063 inc fab8(ic.233*modefreq05.2(*myonega))/(2*myonega))-0.01 P 41 relmoden063 inc fab8(ic.233*modefreq07.1(*myonega))/(2*myonega))-0.01 P 42 relmoden071 inc fab8(ic.233*modefreq07.2(*myonega))/(2*myonega))-0.01 P 44 relmoden071 inc fab8(ic.233*modefreq07.2(*myonega))/(2*myonega))-0.01 P 45 relmoden073 inc fab8(ic.233*modefreq07.2(*myonega))/(2*myonega))-0.01 P 46 relmoden073 inc fab8(ic.233*modefreq07.3(*myonega))/(2*myonega))-0.01 P 47 relimoden073 inc fab8(ic.233*modefreq07.3(*myonega))/(2*myonega))-0.01 P 48 re	34	rei2modeh052	ine	fabs((6.2	332*modefreq052-	-(2*myomega))/(2*myomega))-0.01	
36 rel2modeh051 inc fabs((6.2322*modefreq051-(1*myonega))/(2*myonega)).0.01 Ľ 37 rel7modeh061 inc fabs((6.232*modefreq051-(2*myonega))/(2*myonega)).0.01 Ľ 38 rel2modeh062 inc fabs((6.2332*modefreq052-(1*myonega))/(2*myonega)).0.01 Ľ 41 rel2modeh062 inc fabs((6.2332*modefreq052-(2*myonega))/(1*myonega)).0.01 Ľ 42 rel2modeh053 inc fabs((6.2332*modefreq052-(2*myonega))/(1*myonega)).0.01 Ľ 43 rel1modeh053 inc fabs((6.2332*modefreq072-(2*myonega)).0.01 Ľ 44 rel2modeh071 inc fabs((6.2332*modefreq072-(1*myonega)).(2*myonega).0.01 Ľ 44 rel2modeh072 inc fabs((6.2332*modefreq072-(1*myonega)).(2*myonega).0.01 Ľ 45 rel1modeh073 inc fabs((6.2332*modefreq073-(1*myonega)).(2*myonega).0.01 Ľ 46 rel2modeh073 inc fabs((6.2332*modefreq073-(2*myonega)).0.01 Ľ 47 rel1modeh073 inc fabs((6.2332*modefreq073-(2*myonega)).0.01 Ľ 48 re2modeh073	35	rel1modeh053	ine	fabs((6.2	332*modefreq053-	-(1*myomega))/(1*myomega))-0.01	
37 relfmodeh061 inc fabs((6.2322*modefreq061-(2*myonega))/(2*myonega))-0.01 9 relfmodeh061 inc fabs((6.232*modefreq062-(1*myonega))/(2*myonega))-0.01 Ø relfmodeh062 inc fabs((6.232*modefreq063-(1*myonega))/(2*myonega))-0.01 Ø relfmodeh063 inc fabs((6.232*modefreq063-(1*myonega))/(2*myonega))-0.01 Ø relfmodeh071 inc fabs((6.232*modefreq063-(2*myonega))/(2*myonega))-0.01 Ø relfmodeh071 inc fabs((6.232*modefreq073-(2*myonega))/(2*myonega))-0.01 Ø relfmodeh071 inc fabs((6.232*modefreq073-(2*myonega))/(2*myonega))-0.01 Ø relfmodeh072 inc fabs((6.232*modefreq073-(1*myonega))/(2*myonega))-0.01 Ø relfmodeh073 inc fabs((6.232*modefreq073-(1*myonega))/(2*myonega))-0.01 Ø relfmodeh081 inc fabs((6.232*modefreq073-(1*myonega))/(2*myonega))-0.01 Ø relfmodeh081 inc fabs((6.232*modefreq073-(2*myonega))/(2*myonega))-0.01 Ø relfmodeh081 inc fabs((6.232*modefreq082-(2*myonega))/(2*myonega))-0.01 Ø relfmodeh081 i	36	rel2modeh053	ine	fabs((6.2	832"modefreq053	-(2*myomega))/(2*myomega))-0.01	
38 rel2modeh061 ne fabs((6.2322*modefreq062.(*trymonega))(2*myomega)).0.01 Ľ 40 rel2modeh062 ine fabs((6.2322*modefreq062.(*trymonega))(2*myomega)).0.01 Ľ 41 rel1modeh062 ine fabs((6.232*modefreq063.(*trymonega))(2*myomega)).0.01 Ľ 42 rel2modeh063 ine fabs((6.2332*modefreq063.(*trymonega))(2*myomega)).0.01 Ľ 43 rel1modeh071 ine fabs((6.2332*modefreq071.(*trymonega))(2*myomega)).0.01 Ľ 44 rel2modeh071 ine fabs((6.2332*modefreq072.(*trymonega))(2*myomega)).0.01 Ľ 45 rel1modeh072 ine fabs((6.2332*modefreq073.(*trymonega))(2*myomega)).0.01 Ľ 46 rel2modeh073 ine fabs((6.2332*modefreq073.(*trymonega))(2*myomega)).0.01 Ľ 47 rel1modeh073 ine fabs((6.2332*modefreq073.(*trymonega))(2*myomega)).0.01 Ľ 48 rel2modeh073 ine fabs((6.2332*modefreq03.(*trymonega))(1*myomega)).0.01 Ľ 49 rel2modeh082 ine fabs((6.2332*modefreq03.(*trymonega))(1*myomega)).0.01 Ľ	37	rel1modeh061	ine	fabs((6.2	832"modefreq061	-(1*myomega))/(1*myomega))-0.01	
39 relf modeh062 inc fabs((6.232)*modefreq062.2*myonega)/(2*myonega).0.01 r/ 41 relfmodeh062 inc fabs((6.232)*modefreq063.2*myonega)/(2*myonega).0.01 r/ 42 relfmodeh063 inc fabs((6.232)*modefreq063.2*myonega)/(2*myonega).0.01 r/ 43 relfmodeh071 inc fabs((6.233)*modefreq071.4*myonega)/(2*myonega).0.01 r/ 44 relfmodeh071 inc fabs((6.233)*modefreq071.4*myonega)/(2*myonega).0.01 r/ 45 relfmodeh072 inc fabs((6.233)*modefreq072.4*myonega)/(2*myonega).0.01 r/ 46 relEmodeh073 inc fabs((6.233)*modefreq072.4*myonega)/(2*myonega).0.01 r/ 47 relfmodeh073 inc fabs((6.233)*modefreq072.4*myonega)/(2*myonega).0.01 r/ 48 relEmodeh073 inc fabs((6.233)*modefreq081.4*myonega)/(2*myonega).0.01 r/ 49 rel2modeh081 inc fabs((6.233)*modefreq082.4*myonega)/(2*myonega).0.01 r/ 51 relfmodeh081 inc fabs((6.233)*modefreq083.4*myonega)/(2*myonega).0.01 r/ 52	38	rel2modeh061	ine	fabs((6.2	832*modefreq061	-(2*myomega))/(2*myomega))-0.01	
40 rel2modeh062 ine fabsi(6.2322*modefreq063.(1*myonega))/(2*myonega)).0.01 // 41 rel1modeh063 ine fabsi(6.2322*modefreq063.(1*myonega))/(2*myonega)).0.01 // 42 rel2modeh063 ine fabsi(6.2322*modefreq073.(1*myonega))/(2*myonega)).0.01 // 43 rel1modeh071 ine fabsi(6.2322*modefreq071.(1*myonega))/(1*myonega)).0.01 // 44 rel2modeh071 ine fabsi(6.2322*modefreq072.(1*myonega)).0.01 // 45 rel1modeh073 ine fabsi(6.232*modefreq073.(1*myonega)).0.01 // 46 rel2modeh073 ine fabsi(6.2332*modefreq073.(1*myonega)).0.01 // 47 rel1modeh073 ine fabsi(6.2332*modefreq032.(1*myonega)).0.11 // 48 rel2modeh073 ine fabsi(6.2332*modefreq081.(1*myonega)).0.11 // 50 rel1modeh081 ine fabsi(6.2332*modefreq081.(1*myonega)).0.11 // 51 rel1modeh081 ine fabsi(6.2332*modefreq081.(1*myonega)).0.11 // 52 rel2modeh082 ine	39	rel1modeh062	ine	fabs((6.2	832*modefreq062-	-(1*myomega))/(1*myomega))-0.01	
41 relmodeh063 inc fabs((6.232*modefreq05.2*rymogea))/(2*rymogea)).0.01 relmodeh063 42 relmodeh063 inc fabs((6.232*modefreq07.2*rymogea))/(2*rymogea)).0.01 relmodeh07 43 relmodeh071 inc fabs((6.232*modefreq07.1*(1*myomega))/(2*rnyomega)).0.01 relmodeh07 44 re2modeh072 inc fabs((6.233*modefreq07.2*(1*myomega))/(2*rnyomega)).0.01 re1 45 re1modeh071 inc fabs((6.233*modefreq07.2*(1*myomega))/(2*rnyomega)).0.01 re1 46 re2modeh073 inc fabs((6.233*modefreq07.3*(2*myomega))/(2*rnyomega)).0.01 re1 47 re1modeh073 inc fabs((6.233*modefreq03.2*(1*myomega))/(2*rnyomega)).0.01 re1 48 re2modeh073 inc fabs((6.233*modefreq03.2*rymodefreq03.1*rmyomega)).0.01 re1 49 re1modeh082 inc fabs((6.233*modefreq03.2*rymodefreq03.1*rmyomega)).0.01 re1 51 re1modeh083 inc fabs((6.233*modefreq03.2*rymomega)).0.11 re1 52 re2modeh093 inc fabs((6.233*modefreq03.2*rymodefreq03.2*rymomega)).0.11 re1	40	rel2modeh062	ine	fabs((6.2	832*modefreq062-	-(2*myomega))/(2*myomega))-0.01	
42 relEmodeh03 ine fabsi(6.2832*modefreq071-(1*myonega))(2*myonega).0.01 r/ 43 relEmodeh071 ine fabsi(6.2832*modefreq071-(1*myonega))(2*myonega).0.01 r/ 44 relEmodeh071 ine fabsi(6.2832*modefreq072-(1*myonega))(2*myonega).0.01 r/ 44 relEmodeh071 ine fabsi(6.2832*modefreq072-(2*myonega))(2*myonega).0.01 r/ 45 relEmodeh072 ine fabsi(6.2832*modefreq073-(2*myonega))(2*myonega).0.01 r/ 48 relEmodeh073 ine fabsi(6.2832*modefreq073-(2*myonega))(2*myonega).0.01 r/ 48 relEmodeh073 ine fabsi(6.2832*modefreq081-(2*myonega))(2*myonega).0.01 r/ 49 relEmodeh081 ine fabsi(6.2832*modefreq081-(2*myonega))(2*myonega).0.01 r/ 50 relEmodeh081 ine fabsi(6.2832*modefreq082-(2*myonega))(2*myonega).0.01 r/ 51 relEmodeh083 ine fabsi(6.2832*modefreq082-(2*myonega))(1*myonega).0.01 r/ 52 relEmodeh083 ine fabsi(6.2832*modefreq082-(1*myonega))(1*myonega).0.01 r/ 54 relEmodeh083 ine fabsi(6.2832*modefreq082-(1*myoneg	41	rel1modeh063	ine	fabs((6.2	332*modefreq063-	-(1*myomega))/(1*myomega))-0.01	
43 relf modeh071 inc fabs((6.2832*modefreq071-(2*myonega))(2*myonega)-0.01 relf 44 relfmodeh071 inc fabs((6.2832*modefreq072-(1*myonega))(2*myonega)-0.01 relf 45 relfmodeh071 inc fabs((6.2832*modefreq072-(1*myonega))(2*myonega)-0.01 relf 46 relfmodeh073 inc fabs((6.2832*modefreq073-(1*myonega))(2*myonega)-0.01 relf 47 relfmodeh073 inc fabs((6.2832*modefreq073-(2*myonega))(2*myonega)-0.01 relf 48 re2modeh081 inc fabs((6.2832*modefreq03-(2*myonega))(2*myonega)-0.01 relf 50 relfmodeh082 inc fabs((6.2832*modefreq03-(2*myonega))(1*myonega)-0.01 relf 51 relfmodeh082 inc fabs((6.2832*modefreq03-(2*myonega))(2*myonega)-0.01 relf 52 relfmodeh082 inc fabs((6.2832*modefreq03-(2*myonega))(2*myonega)-0.01 relf 53 relfmodeh031 inc fabs((6.2832*modefreq03-(2*myonega))(2*myonega)-0.01 relf 54 relfmodeh031 inc fabs((6.2832*modefreq03-(2*myonega))(2*myonega)-0.01 relf	42	rel2modeh063	ine	fabs((6.2	332*modefreq063-	-(2*myomega))/(2*myomega))-0.01	
44 relZmodeh071 inc fabs((6.2832*modefreq07-1(2*myonega))(2*myonega)).0.01 r/ 45 relTmodeh072 inc fabs((6.2832*modefreq07-2(1*myonega))(2*myonega)).0.01 r/ 46 relZmodeh072 inc fabs((6.2832*modefreq07-2(1*myonega))(2*myonega)).0.01 r/ 47 relTmodeh073 inc fabs((6.2832*modefreq07-2(2*myonega))(2*myonega)).0.01 r/ 48 relZmodeh073 inc fabs((6.2832*modefreq081-(2*myonega))(2*myonega)).0.01 r/ 49 relZmodeh081 inc fabs((6.2832*modefreq082-(1*myonega))(1*myonega)).0.01 r/ 50 relTmodeh081 inc fabs((6.2832*modefreq082-(1*myonega))(1*myonega)).0.01 r/ 51 relTmodeh081 inc fabs((6.2832*modefreq083-(1*myonega))(1*myonega)).0.01 r/ 52 relZmodeh083 inc fabs((6.2832*modefreq083-(2*myonega))(1*myonega)).0.01 r/ 53 relTmodeh083 inc fabs((6.2832*modefreq081-(1*myonega))(1*myonega)).0.01 r/ 54 relZmodeh083 inc fabs((6.2832*modefreq081-(1*myonega))(1*myonega)).0.01 r/ <t< td=""><td>43</td><td>rel1modeh071</td><td>ine</td><td>fabs((6.2</td><td>832*modefreq071</td><td>-(1*myomega))/(</td><td>1*myomega))-0.01</td><td></td></t<>	43	rel1modeh071	ine	fabs((6.2	832*modefreq071	-(1*myomega))/(1*myomega))-0.01	
45 relfmodeh072 ine fabs((6.2832*modefreq072-(2*myonega))/(2*myonega)-0.01 r/ 46 reZmodeh072 ine fabs((6.2832*modefreq073-(2*myonega))/(2*myonega)-0.01 r/ 47 relfmodeh073 ine fabs((6.2832*modefreq073-(2*myonega))/(2*myonega)-0.01 r/ 48 reZmodeh073 ine fabs((6.2832*modefreq073-(2*myonega))/(2*myonega)-0.01 r/ 49 reZmodeh081 ine fabs((6.2832*modefreq082-(1*myonega))/(1*myonega))-0.01 r/ 50 relfmodeh081 ine fabs((6.2832*modefreq082-(1*myonega))/(1*myonega))-0.01 r/ 51 relfmodeh081 ine fabs((6.2832*modefreq082-(2*myonega))/(2*myonega))-0.01 r/ 52 relZmodeh082 ine fabs((6.2832*modefreq082-(2*myonega))/(2*myonega))-0.01 r/ 53 relfmodeh093 ine fabs((6.2832*modefreq082-(2*myonega))/(2*myonega))-0.01 r/ 54 relZmodeh091 ine fabs((6.2832*modefreq082-(1*myonega))/(2*myonega))-0.01 r/ 55 relTmodeh091 ine fabs((6.2832*modefreq082-(1*myonega))/(2*myonega))-0.01 r/	44	rel2modeh071	ine	fabs((6.2	832*modefreq071-	-(2*myomega))/(2*myomega))-0.01	<u>v</u>
46 rel2modeh072 ine fabs((6.2832*modefreq072-(2*myonega))(2*myonega))-0.01 r/ 47 rel7modeh073 ine fabs((6.2832*modefreq073-(2*myonega))(2*myonega))-0.01 r/ 48 re2modeh073 ine fabs((6.2832*modefreq073-(2*myonega))(2*myonega))-0.01 r/ 49 re2modeh073 ine fabs((6.2832*modefreq081-(2*myonega))(2*myonega))-0.01 r/ 49 re1modeh082 ine fabs((6.2832*modefreq081-(1*myonega))(1*myonega))-0.01 r/ 51 re1modeh082 ine fabs((6.2832*modefreq081-(1*myonega))(1*myonega))-0.01 r/ 52 re2modeh082 ine fabs((6.2832*modefreq081-(1*myonega))(1*myonega))-0.01 r/ 53 re1modeh083 ine fabs((6.2832*modefreq082-(2*myonega))(2*myonega))-0.01 r/ 54 re2modeh091 ine fabs((6.2832*modefreq082-(1*myonega))(1*myonega))-0.01 r/ 55 re1modeh092 ine fabs((6.2832*modefreq082-(1*myonega))(1*myonega))-0.01 r/ 56 re1modeh093 ine fabs((6.2832*modefreq082-(1*myonega))(1*myonega))-0.01 r/ <	45	rel1modeh072	ine	fabs((6.2	832*modefreq072-	-(1*myomega))/(1*myomega))-0.01	¥
47 reltmodeh073 inc fabs((6.2832*modefreq073-(1*myomega))/(2*myomega))-0.01 reltmodeh073 48 reltmodeh073 inc fabs((6.2832*modefreq073-(2*myomega))/(2*myomega))-0.01 reltmodeh081 49 reltmodeh081 inc fabs((6.2832*modefreq082-(1*myomega))/(2*myomega))-0.01 reltmodeh081 50 reltmodeh081 inc fabs((6.2832*modefreq082-(2*myomega))/(2*myomega))-0.01 reltmodeh081 51 reltmodeh081 inc fabs((6.2832*modefreq082-(2*myomega))/(2*myomega))-0.01 reltmodeh083 52 reltmodeh083 inc fabs((6.2832*modefreq082-(2*myomega))/(2*myomega))-0.01 reltmodeh083 53 reltmodeh083 inc fabs((6.2832*modefreq082-(1*myomega))/(2*myomega))-0.01 reltmodeh083 54 reltmodeh091 inc fabs((6.2832*modefreq082-(1*myomega))/(2*myomega))-0.01 reltmodeh092 56 reltmodeh092 inc fabs((6.2832*modefreq082-(1*myomega))/(1*myomega))-0.01 reltmodeh092 57 reltmodeh093 inc fabs((6.2832*modefreq082-(1*myomega))/(1*myomega))-0.01 reltmodeh093 58 reltmodeh093 inc fabs((6.2832*modefreq082-(2*myomega))/(1*myomega))-0.01 reltmodeh093 <td>46</td> <td>rel2modeh072</td> <td>ine</td> <td>fabs((6.2</td> <td>832*modefreq072-</td> <td>-(2*myomega))/(</td> <td>2*myomega))-0.01</td> <td><u>v</u></td>	46	rel2modeh072	ine	fabs((6.2	832*modefreq072-	-(2*myomega))/(2*myomega))-0.01	<u>v</u>
48 re2modeh073 Inc fabs((6.2832*modefreq073-(2*myonega))/(2*myonega))-0.01 re1modeh081 49 re2modeh081 inc fabs((6.2832*modefreq081-(1*myonega))(1*myonega))-0.01 re1modeh082 50 re1modeh082 inc fabs((6.2832*modefreq081-(1*myonega))(1*myonega))-0.01 re1modeh082 51 re1modeh082 inc fabs((6.2832*modefreq083-(2*myonega))(2*myonega))-0.01 re1modeh082 52 re2modeh082 inc fabs((6.2832*modefreq083-(1*myonega))(2*myonega))-0.01 re1modeh083 54 re2modeh083 inc fabs((6.2832*modefreq083-(1*myonega))(2*myonega))-0.01 re1modeh093 55 re1modeh091 inc fabs((6.2832*modefreq082-(1*myonega))(2*myonega))-0.01 re1modeh091 56 re1modeh091 inc fabs((6.2832*modefreq082-(2*myonega))(2*myonega))-0.01 re1modeh091 58 re2modeh093 inc fabs((6.2832*modefreq082-(2*myonega))(2*myonega))-0.01 re1modeh093 59 re1modeh093 inc fabs((6.2832*modefreq082-(2*myonega))(2*myonega))-0.01 re1modeh103 60 re2modeh093 inc fabs((6.2832*modefr	47	rel1modeh073	ine	fabs((6.2	832*modefreq073	-(1*myomega))/(1*myomega))-0.01	r
49 re2modeh081 ine fabs((6.2832*modefreq081-(2*myonega))/(2*myonega))-0.01 re1modeh082 ine fabs((6.2832*modefreq082-(1*myonega))/(1*myonega))-0.01 re1modeh082 ine fabs((6.2832*modefreq082-(1*myonega))/(2*myonega))-0.01 re1modeh081 ine fabs((6.2832*modefreq082-(1*myonega))/(2*myonega))-0.01 re1modeh081 ine fabs((6.2832*modefreq082-(2*myonega))/(2*myonega))-0.01 re1modeh083 ine fabs((6.2832*modefreq082-(2*myonega))/(2*myonega))-0.01 re1modeh083 ine fabs((6.2832*modefreq081-(2*myonega))/(2*myonega))-0.01 re1modeh083 ine fabs((6.2832*modefreq081-(2*myonega))/(2*myonega))-0.01 re1modeh092 ine fabs((6.2832*modefreq081-(1*myonega))/(1*myonega))-0.01 re1modeh092 ine fabs((6.2832*modefreq082-(2*myonega))/(2*myonega))-0.01 re1modeh092 ine fabs((6.2832*modefreq082-(2*myonega))/(2*myonega))-0.01 re1modeh093 ine fabs((6.2832*modefreq082-(2*myonega))/(2*myonega))-0.01 re1modeh093 ine fabs((6.2832*modefreq082-(2*myonega))/(2*myonega))-0.01 re1modeh093 ine fabs((6.2832*modefreq082-(2*myonega))/(2*myonega))-0.01 re1modeh102 ine fabs((6.2832*modefreq010-(2*myonega))/(2*myonega))-0.01 re1modeh102 ine fabs((6.2832*modefreq010-(2*myonega))/(2*myonega))-0.01	48	rel2modeh073	ine	fabs((6.2	832*modefreq073	-(2*myomega))/(2*myomega))-0.01	r
50 rellmodeh081 ine fabs((6.2832*modefreq082.(1*myonega))((1*myonega)).0.01 r/ 51 rellmodeh081 ine fabs((6.2832*modefreq082.(2*myonega))(2*myonega)).0.01 r/ 52 rel2modeh082 ine fabs((6.2832*modefreq082.(2*myonega))(2*myonega)).0.01 r/ 53 rel1modeh083 ine fabs((6.2832*modefreq082.(2*myonega))(2*myonega)).0.01 r/ 54 rel2modeh083 ine fabs((6.2832*modefreq082.(2*myonega))(2*myonega)).0.01 r/ 55 rel2modeh091 ine fabs((6.2832*modefreq082.(1*myonega))(2*myonega)).0.01 r/ 56 rel2modeh091 ine fabs((6.2832*modefreq082.(1*myonega))(2*myonega)).0.01 r/ 57 rel1modeh091 ine fabs((6.2832*modefreq082.(1*myonega))(2*myonega)).0.01 r/ 58 rel2modeh092 ine fabs((6.2832*modefreq082.(2*myonega))(2*myonega)).0.01 r/ 59 rel1modeh093 ine fabs((6.2832*modefreq023.(2*myonega)).0.01 r/ 60 rel2modeh093 ine fabs((6.2832*modefreq101.(*myonega)).0.01 r/ 61	49	rel2modeh081	ine	fabs((6.2	832*modefreq081	-(2*myomega))/(2*myomega))-0.01	v
51 relfmodeh081 inc fabs((6.2832*modefreq08.1(*myomega))((1*myomega)).0.01 Image: Comparison of the comparison of th	50	rel1modeh082	ine	fabs((6.2	332*modefreq082-	-(1*myomega))/(1*myomega))-0.01	¥
52 re2modeh082 ine fabs((6.2832*modefreq08-2(2*myomega))/(2*myomega))-0.01 re1modeh083 ine fabs((6.2832*modefreq08-2(1*myomega))/(2*myomega))-0.01 re1modeh083 ine fabs((6.2832*modefreq08-2(1*myomega))/(2*myomega))-0.01 re1modeh083 ine fabs((6.2832*modefreq08-2(1*myomega))/(2*myomega))-0.01 re1modeh093 ine fabs((6.2832*modefreq08-2(1*myomega))/(2*myomega))-0.01 re1modeh092 ine fabs((6.2832*modefreq08-2(1*myomega))/(1*myomega))-0.01 re1modeh092 ine fabs((6.2832*modefreq08-2(1*myomega))/(1*myomega))-0.01 re1modeh093 ine fabs((6.2832*modefreq08-2(1*myomega))/(1*myomega))-0.01 re1modeh093 ine fabs((6.2832*modefreq08-2(2*myomega))/(2*myomega))-0.01 re1modeh093 ine fabs((6.2832*modefreq03-2(2*myomega))/(2*myomega))-0.01 re1modeh103 ine fabs((6.2832*modefreq03-2(2*myomega))/(2*myomega))-0.01 re1modeh101 ine fabs((6.2832*modefreq10-2(2*myomega))/(1*myomega))-0.01 re1modeh102 ine fabs((6.2832*modefreq10-2(2*myomega))/(1*myomega))-0.01 re1modeh102 ine fabs((6.2832*modefreq10-2(2*myomega))/(1*myomega))-0.01 re1modeh103 ine fabs((6.2832*modefreq10-2(2*myomega))/(1*myomega))-0.01 re1modeh103 ine fabs((6.2832*modefreq10-2(2*myomega))/(1*myomega))-0.01 <td>51</td> <td>rel1modeh081</td> <td>ine</td> <td>fabs((6.2</td> <td>832*modefreq081-</td> <td>-(1*myomega))/(</td> <td>1*myomega))-0.01</td> <td>V</td>	51	rel1modeh081	ine	fabs((6.2	832*modefreq081-	-(1*myomega))/(1*myomega))-0.01	V
53 relimodeh03 inc fabs((6.2322*modefreq083-(1*myonega))/(2*myonega))-0.01 rel 54 re2modeh0403 inc fabs((6.2322*modefreq083-(2*myonega))/(2*myonega))-0.01 rel 55 re2modeh091 inc fabs((6.2322*modefreq081-(2*myonega))/(2*myonega))-0.01 rel 55 re1modeh091 inc fabs((6.232*modefreq081-(1*myonega))/(2*myonega))-0.01 rel 56 re1modeh091 inc fabs((6.232*modefreq082-(2*myonega))/(2*myonega))-0.01 rel 58 re2modeh093 inc fabs((6.232*modefreq082-(2*myonega))/(2*myonega))-0.01 rel 59 re1modeh093 inc fabs((6.232*modefreq082-(2*myonega))/(2*myonega))-0.01 rel 50 re1modeh093 inc fabs((6.232*modefreq012-(2*myonega))/(2*myonega))-0.01 rel 61 re2modeh101 inc fabs((6.232*modefreq102-(1*myonega))/(2*myonega))-0.01 rel 63 re1modeh102 inc fabs((6.232*modefreq102-(1*myonega))/(2*myonega))-0.01 rel 64 re2modeh101 inc fabs((6.232*modefreq102-(2*myonega))/(2*myonega))-0.01 rel	52	rel2modeh082	ine	fabs((6.2	832*modefreq082-	-(2*myomega))/(2*myomega))-0.01	V
54 re2modeh03i ine fabs((6.2832*modefreq08-12*myomega))/(2*myomega))-0.01 re1modeh091 55 re2modeh091 ine fabs((6.2832*modefreq091-(2*myomega))/(2*myomega))-0.01 re1modeh092 56 re1modeh092 ine fabs((6.2832*modefreq092-(2*myomega))/(1*myomega))-0.01 re1 57 re1modeh092 ine fabs((6.2832*modefreq092-(2*myomega))/(1*myomega))-0.01 re1 58 re2modeh093 ine fabs((6.2832*modefreq092-(2*myomega))/(2*myomega))-0.01 re1 59 re1modeh093 ine fabs((6.2832*modefreq093-(2*myomega))/(2*myomega))-0.01 re1 60 re2modeh093 ine fabs((6.2832*modefreq093-(2*myomega))/(2*myomega))-0.01 re1 61 re2modeh101 ine fabs((6.2832*modefreq03-(2*myomega))/(1*myomega))-0.01 re1 62 re1modeh102 ine fabs((6.2832*modefreq102-(1*myomega))/(1*myomega))-0.01 re1 63 re1modeh102 ine fabs((6.2832*modefreq103-(1*myomega))/(1*myomega))-0.01 re1 64 re2modeh103 ine fabs((6.2832*modefreq112-(1*myomega))/(1*myomega))-0.01 re1	53	rel1modeh083	ine	fabs((6.2	832*modefreq083-	-(1*myomega))/(1*myomega))-0.01	V
55 rel2modeh091 Inc fabs((6.2832*modefreq091-(2*myonega))/(2*myonega))-0.01 Important 56 rel1modeh092 Inc fabs((6.2832*modefreq092-(1*myonega))/(1*myonega))-0.01 Important 57 rel1modeh091 Inc fabs((6.2832*modefreq092-(1*myonega))/(1*myonega))-0.01 Important 58 rel2modeh092 Inc fabs((6.2832*modefreq092-(1*myonega))/(1*myonega))-0.01 Important 59 rel1modeh093 Inc fabs((6.2832*modefreq092-(1*myonega))/(1*myonega))-0.01 Important 60 rel2modeh093 Inc fabs((6.2832*modefreq093-(2*myonega))/(2*myonega))-0.01 Important 61 rel2modeh103 Inc fabs((6.2832*modefreq101-(2*myonega))/(2*myonega))-0.01 Important 62 rel1modeh102 Inc fabs((6.2832*modefreq102-(2*myonega))/(2*myonega))-0.01 Important 63 rel1modeh103 Inc fabs((6.2832*modefreq103-(1*myonega))/(1*myonega))-0.01 Important 64 re2modeh110 Inc fabs((6.2832*modefreq103-(2*myonega))/(2*myonega))-0.01 Important 65 re1modeh103 Inc fabs((6.2832*modef	54	rel2modeh083	ine	fabs((6.2	832*modefreq083-	-(2*myomega))/(2*myomega))-0.01	V
56 relfmodeh092 ine fabs((6.2832*modefreq082.(1*myomega))/(1*myomega)).0.01 Image: Constraint of the constraint of t	55	rel2modeh091	ine	fabs((6.2	32*modefreq091	-(2*myomega))/(2*myomega))-0.01	V
57 relf.modeh091 Inc fabs((6.2832*modefreq091.(1*myomega))(1*myomega))-0.01 Image:	56	rel1modeh092	ine	fabs((6.2	32*modefreq092	-(1*myomega))/(1*myomega))-0.01	v
58 rel2modeh092 inc fabs((6.2832*modefreq082.(2*myomega))/(2*myomega)).0.01 Image: Constraint of the constraint of t	57	rel1modeh091	ine	fabs((6.2	32*modefreq091	-(1*myomega))/(1*myomega))-0.01	2
59 relf.modeh033 ine fabs((6.2322*modefreq03-(1*myomega))(2*myomega))-0.01 Image: I	58	rel2modeh092	ine	fabs((6.2	32*modefreq092	-(2*myomega))/(2*myomega))-0.01	2
60 re2modeh033 ine fabs((6.2322*modefreq03-(2*myonega))/(2*myonega))-0.01 Image: Comparison of the company	59	rel1modeh093	ine	fabs((6.2	32*modefreq093	-(1*myomega))/(1*myomega))-0.01	
61 re2modeh101 ine fabs((6.2322*modefreq10.4/2*myomega))/(2*myomega)).0.01 Image: Comparison of the comparison of th	60	rel2modeh093	ine	fabs((6.2	32*modefreq093	-(2*myomega))/(2*myomega))-0.01	2
62 rellmodeh102 ine fabs((6.2832*modefreq102.(1*myomega))(1*myomega))-0.01 Image: Image and the state of t	61	rel2modeh101	ine	fabs((6.2	32*modefreq101	-(2*myomega))/(2*myomega))-0.01	2
63 relfmodeh101 ine fabs((6.2332*modefreq101-(1*myomega))(1*myomega))-0.01 Image: Constraint of the constraint of th	62	rel1modeh102	ine	fabs((6.2	332*modefreq102-	-(1*myomega))/(1*myomega))-0.01	V
64 rel2modeh102 ine fabs((6.2832*modefreq102-(2*myonega))/(2*myonega))-0.01 Image: Image and the state of	63	rel1modeh101	ine	fabs((6.2	32*modefreq101-	-(1*myomega))/(1*myomega))-0.01	V
65 rel1modeh103 ine fabs((6.2832*modefreq103-(1*myonega))/(1*myonega))-0.01 Image: Image and the state of	64	rel2modeh102	ine	fabs((6.2	332*modefreq102-	-(2*myomega))/(2*myomega))-0.01	V
66 rel2modeh103 ine fabs((6.2832*modefreq103.42*myonega))/(2*myonega)).0.01 Image: Constraint of the second sec	65	rel1modeh103	ine	fabs((6.2	B32*modefrea103-	-(1*mvomega))/(1*mvomega))-0.01	V
67 rel2modeh111 ine fabs((6.2832*modefreq112.(1*myomega))/(2*myomega)).0.01 IV 68 rel1modeh112 ine fabs((6.2832*modefreq112.(1*myomega))(1*myomega)).0.01 IV 69 rel1modeh111 ine fabs((6.2832*modefreq112.(1*myomega))(1*myomega)).0.01 IV 70 rel2modeh111 ine fabs((6.2832*modefreq112.(2*myomega))(2*myomega)).0.01 IV 71 rel2modeh113 ine fabs((6.2832*modefreq113.(1*myomega))(2*myomega)).0.01 IV 72 rel2modeh113 ine fabs((6.2832*modefreq113.(2*myomega))(2*myomega)).0.01 IV 73 maxptratio ine 1.36-ptratio IV 74 minptratio ine ptratio-1.34 IV	66	rel2modeh103	ine	fabs((6.2	332*modefreq103-	(2*mvomega))/(2*mvomega))-0.01	V
68 rel1modeh112 ine fabs((6.2832*modefreq112.(1*myomega))(1*myomega)).0.01 Image: the second seco	67	rel2modeh111	ine	fabs((6.2	332*modefreg111	(2*myomega))/(2*myomega))-0.01	V
69 rel1modeh111 ine fabs((6.2832*modefreq111-(1*myomega))(1*myomega))-0.01 Image// image	68	rel1modeh112	ine.	fabs((6.2	B32*modefreg112	(1*mvomega))/(1*mvomega))-0.01	V
70 returneden112 inc fabs((6.2832*modefreq113-(2*myomega))(2*myomega)).0.01 Image/(1 = 10 = 10) 71 retImodeh113 inc fabs((6.2832*modefreq113-(1*myomega))((1*myomega)).0.01 Image/(1 = 10 = 10) 72 retImodeh113 inc fabs((6.2832*modefreq113-(2*myomega))((1*myomega)).0.01 Image/(1 = 10 = 10) 73 maxptratio inc 1.36-ptratio Image/(1 = 10 = 10) 74 minptratio inc ptratio-1.34 Image/(1 = 10 = 10)	69	rel1modeh111	ine	fabs((6.2	32*modefreq111	-(1*myomega))/(1*myomega))-0.01	P
71 reltmodeh113 ine fabs((6.2832*modefreq113-(1*myonega))(1*myonega)).0.01 V 72 rel2modeh113 ine fabs((6.2832*modefreq113-(2*myonega))(1*myonega)).0.01 V 73 maxptratio ine 1.36-ptratio V 74 minptratio ine ptratio-1.34 V	70	rel2modeh112	ine	fabs((6.2	32*modefreg112	-(2*myomega))/(2*myomega))-0.01	P
72 rel2modeh113 inc fabs((6.2832*modefreq113.4/2*myomega))/0.011 Image//0.011 <	71	rel1modeh113	ine	fabs((6.2	32*modefreq113	(1*myomega))/(1*myomega))-0.01	
73 maxptratio inc 1.36-ptratio 74 minptratio inc ptratio-1.34	72	rel2modeh113	ine	fabs((6.2	R32*modefreq113	(2*myomega))/(2*myomena))=0.01	
74 minptratio ine ptratio-1.34 III	73	maxntratio	ine	1003((0.2	11	36-ntratio	2, Smoga/)=0.01	
	74	minntratio	ine		nti	ratio_1.34		
		minpolato			pu			

© 2010 ANSYS, Inc. All rights reserved.

Cancel OK

Cancel

Evolutionary Algorithm





© 2010 ANSYS, Inc. All rights reserved.

Evolutionary Algorithm





Conclusion Optimization



- Sensitivity shows optimization potential
- Pre-Optimization, ARSM, increases quality
- EA leads to further improvement

	Initial	SA	ARSM	EA
Total Pressure Ratio	1.3456	1.3497	1.3479	1.3485
Efficiency [%]	86.72	89.15	90.62	90.67
#Designs	-	100	105	84





© 2010 ANSYS, Inc. All rights reserved.



Design Optimization

Robustness Evaluation

Random Fields



eliability Analysis

Application of stochastic and robustness evaluation





- Analysis models become increasingly detailed
- Numerical procedures become more and more complex
- Substantially more precise data are required for the analysis
- Deterministic optimum design is frequently pushed to the design space boundaries
- Optimized designs lead to high imperfection sensitivities
- Optimized designs tend to loose robustness
- But assessment of design robustness, reliability and safety will be more and more important
- Because of that integration of optimization and stochastic analysis methods will be necessary

What does robustness evaluation mean?





OActual, robustness evaluation means

- O global variance-based sensitivity analysis
- O of computational design models
- O according to random parameters with
- O input correlation and
- specification of random parameter sensitivities for stochastic analysis and

O identification of non-robust behavior.

But in many cases the nonrobustness of

- O computational methods
- O computational software
- O computational hardware

◯is involved.

O ...

Statistical data



- Where to get statistical data?
 - From lab testing (always best)
 - **O** From measurements of suppliers/manufacturers
 - From technical references (mostly only mean values provides)
 - Estimation mostly Gaussian or lognormal distribution standard deviation of about ±10%
- Typical scatter for metallic materials
 - Young's modulus Gaussian with a standard deviation of about ±3-5%
 - Shear modulus Gaussian with a standard deviation of about ±4-8%
 - Poisson's ratio Lognormal with a standard deviation of about ±10-20%
 - Density Gaussian with a standard deviation of about ±1-6%
 - Thermal expansion coeff. Gaussian with a standard deviation of about ±4-5%
 - Heat conductivity Gaussian with a standard deviation of about ±3-5%
 - Heat capacity Gaussian with a standard deviation of about ±5-7%
 - Yield strength Rp0.2 Lognormal with a standard deviation of ±5%



Uncertainties and Tolerances

Property	SD/Mean %
Metallic materiales, yield	15
Carbon fiber composites, rupture	17
Metallic shells, buckling strength	14
Junction by screws, rivet, welding	8
Bond insert, axial load	12
Honeycomb, tension	16
Honeycomb, shear, compression	10
Honeycomb, face wrinkling	8
Launch vehicle , thrust	5
Transient loads	50
Thermal loads	7.5
Deployment shock	10
Acoustic loads	40
Vibration loads	20



ANS

- Design
- Material, geometry, loads, constrains,...
- Manufacturing
- Operating processes (misuse)
- Resulting from Deterioration

[Klein, Schueller et.al. Probabilistic Approach to Structural Factors of Safety in Aerospace. Proc. CNES Spacecraft Structures and Mechanical Testing Conf., Paris 1994]

Exceedance Probability



 Probability of reaching values above a threshold for Gaussian distribution



Benefit of robustness evaluation



Oldentification of non-robustness

- O variance of the responses increases
- O moving of the mean values
- O responses exceed the limit states
- **O** undesired outliers
- O system failures

(structural buckling, system resonances, etc.)

Identification the most-relevant random parameters to reduce the stochastic problem
 Estimation of the Sigma Level





Random variables of robustness evaluation



Opti	Robust Outp	ut Strings Co	onstraints Obje	ctives						
#	Name	Distribution	Mean	CoV	Stddev	Lower Cut	Upper	For	A	C
1	Ttin	Normal	313.0	0.0050	1.565	-	-	%	\mathbf{V}	
2	myAirCP	Normal	1004.4	0.0050	5.022	-	-	%	\mathbf{V}	
3	myAirR	Normal	287.1	0.0050	1.435500000000	-	-	%	\mathbf{V}	
4	mymassin	Normal	72.6	0.0050	0.363	-	-	%	\mathbf{V}	
5	myomega	Normal	699.76	0.0050	3.4988	-	-	%	\mathbf{V}	
6	ptin	Normal	1724000.0	0.0050	8620.0	-	-	%	\mathbf{V}	
7	InletWidth	Normal	53.61366106578	0.005000	0.268068305328	-	-	%	\mathbf{V}	
8	ExitWidth	Normal	27.8049298398772	0.0050	0.139024649199	-	-	%	\mathbf{V}	
9	Rimpeller	Normal	292.56	0.0050	1.4628	-	-	%	\mathbf{V}	
10	HubBeta1	Normal	-52.5	-0.0050	0.2625	-	-	%	\mathbf{V}	
11	HubBeta2	Normal	-25.0	-0.0050	0.125	-	-	%	\mathbf{V}	
12	HubBeta3	Normal	-27.01713251922	-0.0050	0.135085662596	-	-	%	\mathbf{V}	
13	ShdBeta1	Normal	-60.26762316109	-0.0050	0.301338115805	-	-	%	\mathbf{V}	
14	ShdBeta2	Normal	-45.0	-0.0050	0.225	-	-	%	\mathbf{V}	
15	ShdBeta3	Normal	-30.0	-0.0050	0.15	-	-	%	\mathbf{V}	
16	HubThk1	Normal	1.0	0.0050	0.0050	-	-	%	\mathbf{V}	
17	HubThk2	Normal	5.9196	0.0050	0.029598	-	-	%	\mathbf{V}	
18	ShdThk1	Normal	1.0301	0.0050	0.005150500000	-	-	%	\mathbf{V}	
19	ShdThk2	Normal	6.0	0.05	0.30000000000	-	-	%	\mathbf{V}	
20	RVHubThk1	Normal	45.0	0.05	2.25	-	-	%	\mathbf{V}	
21	RVHubBeta1	Normal	66.0	0.05	3.30000000000	-	-	%	\mathbf{V}	
22	RVShdBeta1	Normal	62.85486468353	0.05	3.142743234176	-	-	%	\mathbf{V}	
23	RVShdThk1	Normal	45.0	0.05	2.25	-	-	%	\mathbf{V}	
24	Density	Normal	7850.0	0.05	392.5	-	-	%	\mathbf{V}	
25	Youngs_Modulus	Normal	2.0E11	0.05	1.0E10	-	-	%	\mathbf{V}	
26	Poissons_Ratio	Log-Normal	0.3	0.1	0.03	-	-	%	M	

Cancel OK



Random Fields: Background



What is a random field?

- A random function, defined on a structure, which takes random values at any location. One outcome is called realization, the set of all realizations is called ensemble.
- Stochastic properties at each point are defined by stochastic moments (mean, standard deviation ...) and distribution type.
- Dependency between different locations is defined by the correlation function.



Random Fields: Background



Random Fields Simulation:

- Measured Data
- Decomposition of the Covariance Matrix, the Random Field is expanded as a series of deterministic Shape Functions and Random Amplitudes





The correlation coefficient function is a function of the distance between two points. It is characterized by the correlation length



The correlation function must be positive semi-definite. Examples: exponential, triangular.

Theoretical Basics



Special cases:

- Normal distribution type: the random field is characterized completely by mean and covariance function
- Homogeneity: same stochastic properties at any point throughout the structure
- Isotropy: correlation depends on the distance between two points, not the direction
- Zero means: correlation function and covariance function are identical



Measure Spatial Deviations

ANSYS® dynando

- Using 3D digitizer

- Based on the principle of triangulation, projected fringe patterns are observed with cameras, 3D coordinates for each camera pixel are calculated, a polygon mesh of the object's surface is generated

- Deviations from CAD geometry can be calculated







Modeling Spatial Deviations





Random field modeling







Imperfection modes of the random field





Data Reduction



Truncation of random field expansion

- · Eigenvalues are sorted in (usu. strongly) decreasing order
- Highest eigenvalues contribute most to total variance
- → Neglect variables with minor contribution



Data Reduction



• Quality of truncated series: variability fraction = 91%

$$Q = \frac{\sum_{i=1}^{N_{\lambda}} \lambda_i}{\operatorname{trace}(\mathbf{C}_{XX})} \equiv \frac{\sum_{i=1}^{N_{\lambda}} \lambda_i / \operatorname{dim}(\mathbf{C}_{XX})}{\operatorname{trace}(\mathbf{C}_{XX}) / \operatorname{dim}(\mathbf{C}_{XX})}$$

After mesh reduction: normalize variability to number of data



Realizations of the random field

ANSYS[®] dynando







Process integration




Robustness Total Temperature





No criterion of failure for Θ_T

P rel = 0.0512821

x rel = 1.1009

P fit =

x fit =

Probability P(X < x) = 0.95

0.0226627

1.10116

Robustness Total Pressure





© 2010 ANSYS, Inc. All rights reserved.

Robustness Total Pressure





Modification of omega leads to more Robust Design



Robustness Efficiency



OUTPUT: mveta **Most relevant Parameter** ഹ 17 Fitted PDF Coefficients of Prognosis (using MoP) full model: CoP =`65 % 50 Histogram Ξ Limit line INPUT: HubBeta2 ഹ 0 % 12 PDF 100 INPUT: ShdBeta3 9 0 % parameter ഹ **INPUT: Density** 1 % 50 INPUT: HubBeta1 4 4 % ഹ INPUT Ñ INPUT: RVShdBeta1 15 % 0 0.885 0.89 0.895 0.9 0.905 **INPUT: RImpeller** \sim OUTPUT: myeta 21 % Statistic data INPUT: RVHubBeta1 Max: 0.9078 0.8841 Min: 29 % Sigma: 0.003851 Mean: 0.9018 20 40 60 80 100 CoP [%] of OUTPUT: myeta CV: 0.00427 -2.212 Kurtosis: 9.204 Skewness: Fitted PDF: Extreme Typ III (Min) Weibull **Tolerance limit η>90%** Mean: 0.9018 Sigma: 0.003851 Upper cut: 0.9078 ~28% outside Limit x = 0.90.168831 P rel = P fit = 0.277155

© 2010 ANSYS, Inc. All rights reserved.

Robustness Efficiency



Modification of RImpeller and RVHubBeta1 leads to more Robust Design

ANSYS[®]

Robustness Eigen Frequency Mode 1 Harmonic Index 0





Conclusion Robustness Analysis

Non robust behavior with respect to O Efficiency

O Total pressure

- OBut acceptable failure probability level for structural risk
 - O Estimation of a Six Sigma Design

OEfficiency: myeta

- RVHubBeta1 as largest as possible
- O RVShdBeta1 as largest as possible
- O RImpeller as smallest as possible

OTotal pressure: ptratio

- O myomega as largest as possible
- O RImpeller as largest as possible
- Optratio mean -> 1.355



INEQUAL: minptratio vs. INPUT: myomega, (linear) r = 0. INEQUAL: maxptratio vs. INPUT: myomega, (linear) r = -0.709

ANSYS

Successive Robust Design Optimization



ANSYS° dynando

- iterative decoupled loop approach
- in combination with identification of the most significant random and design variables using the multivariate statistic
- first step the robustness evaluation can be used to prove the predictive capability of the simulation model and to
- identify the most important parameters to solve reliability analysis, efficiently
- it is neccessary to evaluate robustness and safety of the design

Process Integration

Sensitivity Analysis

Design Optimization II Robustness Evaluation II

<u>dynando</u>

1



Random Fields

eliability Analysis

Design Optimization II

Opti	Robust Ou	tput Strings	Constraints	Objectives					
#	Name	Value	Ref.Value	Lower Bound	Upper Bound	Туре	Format	Active	Const
1	myomega	699.76	699.76	699.0	703.0	continuous	%20.14f		
2	InletWidth	53	53.6136610657	52.5	57.5	continuous	%20.14f	V	
3	ExitWidth	26	27.8049298398	26.5	28.5	continuous	%20.14f		
4	Rimpeller	305	292.556879245	291	300	continuous	%20.14f	V	
5	HubBeta1	-48	-52.5	-55	-49.5	continuous	%20.14f		
6	HubBeta3	-25	-27.017132519	-28	-26.5	continuous	%20.14f	V	
7	ShdBeta1	-55	-60.267623161	-60.5	-59.5	continuous	%20.14f	V	
8	RVHubThk1	45	45.0	35	66.0	continuous	%20.14f		
9	RVHubBeta1	60	66.0	62.0	68	continuous	%20.14f		
10	RVShdBeta1	60	62.8548646835	60.0	64.0	continuous	%20.14f		
11	RVShdThk1	45	45.0	35.0	55.0	continuous	%20.14f	V	
12	HubBeta2	-25	-25.0	-27.5	-22.5	continuous	%20.14f		V
13	ShdBeta2	-45	-45.0	-49.5	-40.5	continuous	%20.14f		V
14	ShdBeta3	-30	-30.0	-33.0	-27.0	continuous	%20.14f		V
15	HubThk1	1	1.0	0.8	1.2	continuous	%20.14f		V
16	HubThk2	6	5.91963645103	5.0	7.0	continuous	%20.14f		~
17	ShdThk1	1	1.03011230706	0.8	1.2	continuous	%20.14f		
18	ShdThk2	6	6.0	5.0	7.0	continuous	%20.14f		
19	ImpellerBlades	20	20	18.0	24.0	continuous	%20.14f		
20	RVBlades	24	24	21.6	28.7999999999	continuous	%20.14f		

Cancel

OK

ANSYS[®] dynando

Design Optimization II: ARSM





Design Optimization II: ARSM





#Designs

62

100

105

84

Process Integration

Sensitivity Analysis

Design Optimization II

Robustness Evaluation II

Random Fields



eliability Analysis

ANSYS®

12

Robust evaluation II: LHS





Tolerance limit η<90% ~8% outside Tolerance limit Π_T>1.36 ~17% outside

Process Integration

Sensitivity Analysis

Design Optimization III

Robustness Evaluation III

Random Fields



eliability Analysis

ANSYS®

1 AS TI

Design Optimization III: ARSM



ANSYS[®]

Design Optimization III: ARSM





	Initial	SA	ARSM I	EAI	ARSM II	ARSM III
Total Pressure Ratio	1.3456	1.3497	1.3479	1.3485	1.356	1.351
Efficiency [%]	86.72	89.15	90.62	90.67	90.76	90.73
#Designs	-	100	105	84	62	40

© 2010 ANSYS, Inc. All rights reserved.

Process Integration

Sensitivity Analysis

Design Optimization III

Robustness Evaluation III

Random Fields



eliability Analysis

ANSYS®

12W

Robust evaluation III: LHS





Tolerance limit η<90% ~4.5% outside

Robust Design

Tolerance limit 1.4<∏_T<1.36 ~6% outside

Robust evaluation III: Eigen



Safety Design?



Six Sigma Analysis





Calculation of probabilities much lower than E-2 needs detailed know how of all relevant uncertainties and reliability analysis

Sigma level	Varation	Probability of failure	Defects per million (short term)	Defects per million (long term – $\pm 1.5\sigma$ shift)
$\pm 1\sigma$	68.26	3.1 E-1	317,400	697,700
$\pm 2\sigma$	95.46	4.5 E-2	45,400	308,733
$\pm 3\sigma$	99.73	2.7 E-3	2,700	66,803
$\pm 4\sigma$	99.9937	6.3 E-5	63	6,200
$\pm 5\sigma$	99.999943	5.7 E-7	0.57	233
$\pm 6\sigma$	99.9999998	2.0 E-9	0.002	3.4

Methods of Reliability Analysis

ANSYS° dynando

 First and second order reliability method (FORM/SORM)

Sigma level ≥ 2 , $n \leq 50$

- Monte-Carlo-Simulation (MCS)
 - Sigma level ≤ 2 , independent of *n*
- Latin hyper cube sampling (LHS)
 - Sigma level \leq 3, independent of *n*
- Importance sampling using design point (ISPUD)
 - Sigma level \geq 2, $n \leq$ 50
- Adaptive importance sampling (AIS)
 - Sigma level \geq 2, $n \leq$ 15
- Directional sampling (DS)

Sigma level ≥ 2 , $n \leq 20$

- Adaptive response surface method (ARSM)
 - Sigma level \geq 2, $n \leq$ 15

$$P(F) = P[\mathbf{X} : g(\mathbf{X}) \le 0] = \int \dots \int f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$









MLS approximation of g MLS approximation of g 324 324 250 200 150 250 100 50 200 300 -50 150 100 250 50 200 0 -50 150 9 100 50 -0 -4 -2 -2 0 x1

Himmelblau function

-4

-2

x1

Nonlinear two dimensional state function $g(x_1,x_2)$

2

Nonlinear limit state function g(x1,x2)=0

n

Three separated domains with high failure probability density

© 2010 ANSYS, Inc. All rights reserved.

2

0

2

4

9

-6

X2

2

2



0





- Adaptive response surface method
- Directional sampling on MLS
- Design evaluations: 58
- PF = 1.67E-06 (1.99E-06)

- Sigma level independent
- n ≤ 20
- Multiple adaptive DOE
- Supports multi-domain limit states

Adaptive response surface approximation



🔿 Reli	ability settin	gs (🔴
Load/Save Presets		
reliability algorithm	Adaptive r	esponse surface 🔹 🔻
Parameters		
Assumed failure p	orobability	3.4e-6
Samplin	ng method	directional sampling 🗨
Number	of directions	adaptive sampling directional sampling
Initial Do	DEschema	D-optimal quadratic 💌
Initial axial	multiplier	1.0
Following Dol	Eschemes	D-optimal linear 🔻
Rotate Dol	Eschemes	K
Maximum number o	of clusters	3
Max. number of	adaptions	6
Accuracy of failure prob	ability [%]	50.0
Limit bound of parameter ch	anges [%]	2.0
Reset		Cancel OK

- Sampling methods on the MLS approximation:
 - Adaptive Sampling
 - Directional Sampling
 - supports more than two failure domains
 - and sigma level independent
 - Cluster analysis to detect number of failure domains with high failure probability
- Rotatable adaptive designs of experiments to improve the approximation accuracy

Methods of Reliability Analysis

Recommended Areas of Application



*¹only applicable within continuously differentiable response functions

ANSYS[®]

ANSYS[®]



ANSYS[®] dunando







Summary



- Parametric Workflow management
- Automatic and embedded solution
- Parallel and distributed solver runs
- Process integration within optiSLang
- Efficient Robust Design Optimization with
- Quadratic convergence rate and
- 18 design parameters and
- 26 random geometry parameters,



- Optimized robust design: 5% improvement of the efficiency (η<90%, failure rate ~4.5%) Tolerance limit (1.4<Πτ<1.36, failure rate ~6%)
- Optimized Six Sigma design $P(\mathcal{F}) \approx 3 \cdot 10^{-7}$
- N = 100 + 105 + 84 + 100 + 62 + 50 + 40 + 50 + 68 = 659 design evaluations (SA)(EA)(ARSM)(RE)(ARSM)(RE)(ARSM)(RE)(RA)
- Calculation time: 10 days (8 CPUs)

