Aggressive Design in Turbomachinery

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Introduction
Uncertainty Quantification (UQ)

• UQ is not just about an error bar

• It is a rapidly developing field encompassing
  • CFD prediction
  • Meshing and geometry generation and processing
  • Algorithms for efficient sensitivity analysis
  • Computationally tractable frameworks for robust design
  • Statistical analysis on sparse data

• Must be factored when designing with models for engine
  • Uncertainties (variability) exist in both models & engine

• Goal of UQ research in CFD methods is to
  • Increase engine efficiency given variability
  • Maintain engine efficiency given variability
Engine Aleatory Uncertainties

- Inlet lip manufacturing uncertainties
- Ice accretion uncertainty in compressors
- Fan rear seal leakage flows
- Tip clearance variations in HP compressors
- Uncertain inlet boundary conditions
- Using GOM data to characterize surface uncertainties
- Unsteady aero uncertainties
- Combustion UQ
- Aero-acoustics UQ in fans
- HP-NGV manufacturing variations on capacity
- Squealer tip thermal deterioration uncertainty
Engine Epistemic Uncertainties

- Surface roughness models in RANS
- Transition modeling
- Hybrid RANS-LES approaches
- Structured/unstructured meshing techniques
- Bayesian hybrid modeling for experimental data – CFD validation
- Conjugate heat transfer modeling uncertainties
- RANS turbulence modeling uncertainties

Mesh independence during optimization
Design Methodology with Uncertainty

Rolls-Royce CFD Methods

3D Designs

Optimization: SOFT
Uncertainty Quantification: SOFT+UQ
Grid & Geometry Generation: PADRAM
CFD Solution: HYDRA
Mathematical Formulation

- Optimization under uncertainty

\[ f = f(s, \omega) \]

Physical model output  \quad \text{design variables}  \quad \text{uncertain variables}

\[ \text{minimize } s \quad f(s, \omega) \]
Mathematical Formulation

- Optimization under uncertainty

\[ f = f(s, \omega) \]

- Methods for optimization under uncertainty
  - Robust design, reliability based design optimization, first order reliability method, second order reliability method, most probable point,…
  - Some methods optimize moments, others optimize tails.
Mathematical Formulation

\[ \minimize_s f(s, \omega) \]

optimization goal is a random function

\[ \minimize_s \mathbb{E}\{f(\omega, s)\} \]

Take the expectation
Mathematical Formulation

Scalarization

Multi-objective optimization

optimization goal is a random function

Take the expectation

optimization goals are deterministic functions
Some Issues

• Scalarization requires apriori knowledge

• Cost of Multi-Objective Optimization
  • Requires many objective function evaluations
  • Order of magnitude more expensive than single-objective problem

• What if a large variance is permissible (the PDF has a favorable skew)?
  • Skewness is not factored in robust design
  • Mean and variance do not uniquely define a PDF

• What if mean and variance are correlated?

• Challenging to optimize for a certain tail probability

These issues motivate the present work
Mathematics of Aggressive Design
Aggressive design seeks to minimize the “distance” between the **current design** PDF and the **target design** PDF.
Aggressive design seeks to minimize the “distance” between the current design PDF and the target design PDF.
**Mathematical Formulation**

\[ f = f(s, \omega) \]

- **physical model output**
- **design variables**
- **uncertain variables**

Designer specifies target pdf of output

\[ t = t(f) \geq 0, \quad \int t(f) \, df = 1 \]

For a fixed design \( s \), uncertainty produces pdf of \( f \)

\[ u_s = u_s(f) \geq 0, \quad \int u_s(f) \, df = 1 \]
Goal is to find the design so that model pdf is as close as possible to the designer’s target.

\[
\min_{s} \delta(t, u_s)
\]

where delta is a distance metric. We choose

\[
\delta(t, u_s) = \int (t - u_s)^2 \, df
\]

because it is differentiable.
Discretize the ‘distance’

Choose an integration rule in the range of the model output

\[
\hat{\delta} \approx \tilde{\delta} = \sum_{i=1}^{M} \left( t(f_i) - u_s(f_i) \right)^2 w_i
\]

\[
= (t - u_s)^T W (t - u_s)
\]

where \( W \) is a diagonal matrix of integration weights, \( t \) is a vector of target pdf evaluations, and \( u_s \) is a vector of model pdf evaluations.
Kernel density estimation is a statistically well-known alternative to histograms

Idea is to replace discrete bins with a unimodal kernel function to obtain an analytic definition for a PDF
Quick detour: Kernel Density Estimation (KDEs)

\[ f(x) = \frac{1}{nh} \sum_{i=1}^{n} K \left( \frac{x - x_i}{h} \right) \]

- \( x_i \) – random samples
- \( h \) – bandwidth
- \( K \) – kernel function

Each sample \( x_i \) is represented by a kernel function with zero mean and a finite variance.

Given a quantity of interest, KDEs can be used to estimate the underlying probability density function.
Quick detour: Kernel Density Estimation (KDEs)

\[ f(x) = \frac{1}{nh} \sum_{i=1}^{n} K \left( \frac{x - x_i}{h} \right) \]

- \( x_i \) – random samples
- \( h \) – bandwidth
- \( K \) – kernel function

Then we sum up all the individual kernels to get the kernel density estimate.

Probability

Kernel density estimate

Quantity of Interest
Kernel density estimation of model PDF

Choose a discretization in the random space (e.g., Monte Carlo samples)

\[ f_j(s) = f(s, \omega_j) \]

Use a kernel density estimate of the model pdf with kernel \( K = K_h \) with bandwidth parameter \( h \)

\[ u_s(f_i) \approx \frac{1}{N} \sum_{j=1}^{N} K(f_j(s) - f_i) \]

- model eval’d at point in uncertain space
- numerical integration points for objective
Choose a discretization in the random space (e.g., Monte Carlo samples)

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- model eval’d at point in uncertain space
- numerical integration points for objective

In vector notation with ‘e’ a vector of ones,

\[ u_s \approx K_s e, \quad K_s(i, j) = \frac{1}{N} K(f_j(s) - f_i) \]
Discrete optimization

\[ \minimize_{s} \ (t - K_s e)^T W (t - K_s e) \]

target pdf

diagonal matrix of integration weights

kernel density estimate
Accelerating the Process

\[ \tilde{\delta}(s) = (t - K_s e)^T W (t - K_s e) \]

Can we use gradient information?

YES!
Gradient of the Objective

\[ \tilde{\delta}(s) = (t - K_s e)^T W (t - K_s e) \]

Use the gradient of \( f \) with respect to design variables \( s \) to compute the gradient of the objective with respect to \( s \).

\[ \nabla_s \tilde{\delta}(s) = 2(t - K_s e)^T W K'_s F' \]

Derivative of kernel

\[ K'_s(i, j) = \frac{1}{N} K'(f_j(s) - f_i) \]

Partials of model with respect to design variables evaluated at points in uncertain space

\[ F' = \begin{bmatrix} \frac{\partial f_1}{\partial s_1} & \cdots & \frac{\partial f_1}{\partial s_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_M}{\partial s_1} & \cdots & \frac{\partial f_M}{\partial s_m} \end{bmatrix} \]

These are obtained as part of the CFD solution using adjoints
Computational Airfoil Design Using Aggressive Design
Computational airfoil design

\[ M_\infty = 0.6752 \]

Inlet mach number for airfoil

Airfoil with Hicks-Henne bump functions

\[ f(w, s) = \text{L/D of airfoil} \]
Computational airfoil design under uncertainty

Mach number PDF

$\mu \left( \frac{\text{Lift}}{\text{Drag}} \right), \; \sigma^2 \left( \frac{\text{Lift}}{\text{Drag}} \right), \; K_{\text{se}}$

Response surface (4 CFD runs)

Sample response surface with Mach number distribution (Stochastic Collocation)

Kernel density estimate
Robust Design Approach

Values of interest:

\[ \mu \left( \frac{\text{Lift}}{\text{Drag}} \right), \quad \sigma^2 \left( \frac{\text{Lift}}{\text{Drag}} \right) \]

Optimization problem:

minimize \[ s \quad \mu \left( \frac{\text{Lift}}{\text{Drag}} \right)^{-1}, \]

\[ \sigma^2 \left( \frac{\text{Lift}}{\text{Drag}} \right) \]

Result:
Aggressive Design Approach

Values of interest: $K_s e, \ t$

Optimization problem: $\min_{s} (t - K_s e)^T W (t - K_s e)$

Result:
Aggressive Design Approach

### Metrics

<table>
<thead>
<tr>
<th></th>
<th>Aggressive design</th>
<th>Robust design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimization Iterations</td>
<td>8</td>
<td>35 (generations)</td>
</tr>
<tr>
<td>Adjoint CFD</td>
<td>21 x 8 (4 lift, 4 drag)</td>
<td>-</td>
</tr>
<tr>
<td>Euler CFD</td>
<td>21 x 4</td>
<td>3500 x 4</td>
</tr>
<tr>
<td>Wall-clock time</td>
<td>37.7 minutes</td>
<td>~1 day</td>
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</tbody>
</table>
We are not making a direct comparison between aggressive design and robust design – one is a single objective problem the other is a multi-objective one

Like comparing apples with oranges

What we are presenting is a new approach for design under uncertainty – based on a target design that the designer has selected

Aggressive design is a simple, single-objective method with a smooth objective function

It leverages gradient information when present
Future Work: Multivariate aggressive design

The present framework extends nicely to multiple quantities of interest
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Aggressive Design Literature

Seshadri, P., Parks, G.T., Shahpar, S., *Aggressive Design and Active Subspaces For Robust Redesign in Turbomachinery*, (Abstract accepted) ASME Turbo Expo 2015, Montreal, Canada

