6. Dresdner Probabilistic-Workshop 2013, 10th/11th October 2013

Enhanced Quadrature Approach applied to a Robust Compressor and Turbine Blade Design

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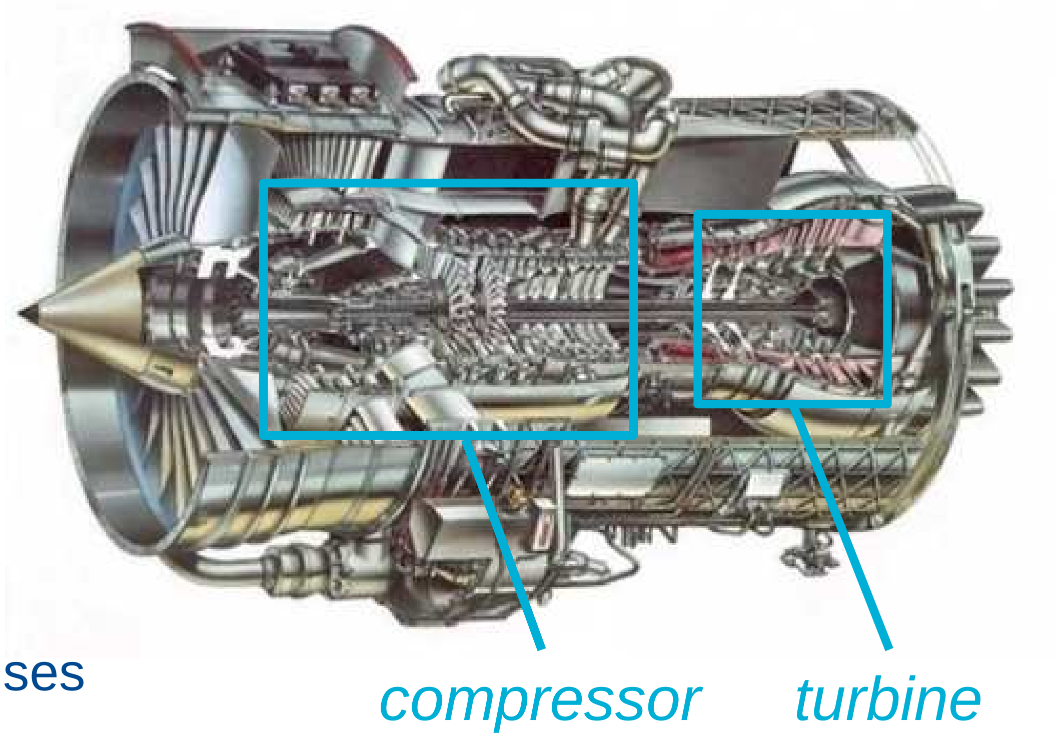
Motivation

- Problem:
  - Deterministic optimization does not feature manufacturing noise and degradation
- Solution:
  - probabilistic simulation describing variations of input parameters and corresponding system responses



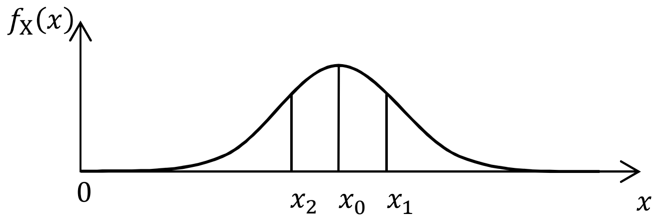
- Opportunity :
  - Monte Carlo Simulation (DS, LHC, oLHC) → very expensive
  - alternativ approach: **Univariate Reduced Quadrature** (URQ) [1]

[1] M. Padulo, M.S. Campobasso, and M.D. Guenov. Novel Uncertainty Propagation Method for Robust Aerodynamic Design. *AIAA Journal*, 49(3):530-543, 2011.

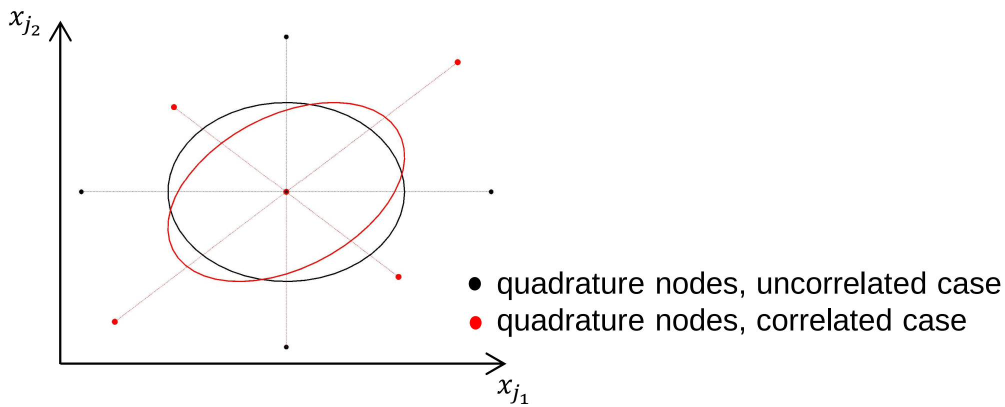
Introduction to the URQ approach

- Subject: Finding an alternative approach, in comparison to expensive Monte Carlo Simulations to save calculation time for estimating $\mu_g = \mathrm{E}[g(x)]$ and $\sigma_g^2 = \mathrm{Var}[g(x)]$
- Basic Idea: Choose a deterministic approach that's based on a Gaussian-Quadrature with 3 nodes per dimension n_d

$$x_0 = \mu_X$$
$$x_1 = \mu_X + h_1\,\sigma_X$$
$$x_2 = \mu_X + h_2\,\sigma_X$$



$$\boldsymbol{x}_0 = \boldsymbol{\mu}_X$$
$$\boldsymbol{x}_{1_i} = \boldsymbol{\mu}_X + h_{1_i}\,\boldsymbol{C}\,\boldsymbol{e}_i$$
$$\boldsymbol{x}_{2_i} = \boldsymbol{\mu}_X + h_{2_i}\,\boldsymbol{C}\,\boldsymbol{e}_i$$



Build the density function $f_X(\boldsymbol{x})$ of the uncertain input parameter $\boldsymbol{x}$ with 3 deterministic chosen nodes $\boldsymbol{x}_0$, $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ for each dimension

Introduction to the URQ approach

- **Idea of URQ:**
  - build $2n_d + 1$ nodes $\boldsymbol{x}$ with corresponding weights $\boldsymbol{w}$

$$\boldsymbol{x}_0 = \boldsymbol{\mu}_X$$
$$\boldsymbol{x}_{1_i} = \boldsymbol{\mu}_X + h_{1_i}\,\sigma_{X_i}\,\boldsymbol{e}_i$$
$$\boldsymbol{x}_{2_i} = \boldsymbol{\mu}_X + h_{2_i}\,\sigma_{X_i}\,\boldsymbol{e}_i$$

σ_X … standard deviation of $\boldsymbol{X}$

$h_{1,2}$ … scalingparameter

  - propagate each node through the response function $g(\boldsymbol{x})$ of interest
  - build mean of system response by

$$\mu_g^{(URQ)} = w_0^{(\mu)} g(\boldsymbol{x}_0) + \sum_{i=1}^{n_d} \left\{ w_{1_i}^{(\mu)} g(\boldsymbol{x}_{1_i}) + w_{2_i}^{(\mu)} g(\boldsymbol{x}_{2_i}) \right\}$$

➡ Calculation cost of $2n_d + 1$ calculations

? **Question** ?

How to find optimal weights $\boldsymbol{w}$ and scaling parameters $\boldsymbol{h}$

Introduction to the URQ approach

- **Solution:** comparison of coefficients between Gaussian-Quadrature and Taylor-series expansion

$$\mu_g^{(URQ)} = w_0^{(\mu)} g(\boldsymbol{x}_0) + \sum_{i=1}^{n_d} \left\{ w_{1_i}^{(\mu)} g(\boldsymbol{x}_{1_i}) + w_{2_i}^{(\mu)} g(\boldsymbol{x}_{2_i}) \right\}$$

$$g(\boldsymbol{x}_0) = g(\boldsymbol{\mu}_X)$$

$$g(\boldsymbol{x}_{1_i}) = g(\boldsymbol{\mu}_X) + \sum_{j=1}^{\infty} \frac{\partial^j g}{\partial x_i^j} \frac{(h_{1_i})^j}{j!}$$

$$g(\boldsymbol{x}_{2_i}) = g(\boldsymbol{\mu}_X) + \sum_{j=1}^{\infty} \frac{\partial^j g}{\partial x_i^j} \frac{(h_{2_i})^j}{j!}$$

Gaussian-Quadrature

$$\mu_g^{(URQ)} = w_0^{(\mu)} g(\boldsymbol{\mu}_X) + \sum_{i=1}^{n_d} \left[g(\boldsymbol{\mu}_X) \left(w_{1_i}^{(\mu)} + w_{2_i}^{(\mu)} \right) + \sum_{k=1}^{\infty} \frac{\partial^k g}{\partial x_i^k} \frac{w_{1_i}^{(\mu)} (h_{1_i})^k + w_{2_i}^{(\mu)} (h_{2_i})^k}{k!} \right]$$

comparison of coefficients

Taylor-series expansion till 4th order

$$\mu_g = E[g(\boldsymbol{x})] = \int_{-\infty}^{+\infty} g(\boldsymbol{x})\, f_X(\boldsymbol{x})\, d\boldsymbol{x}$$

$$= g(\boldsymbol{\mu}_X) + \frac{1}{2} \sum_{i=1}^{n_d} \left(\frac{\partial^2 g}{\partial x_i^2} \right) \sigma_{x_i}^2 + \frac{1}{6} \sum_{i=1}^{n_d} \left(\frac{\partial^3 g}{\partial x_i^3} \right) \gamma_{x_i} \sigma_{x_i}^3 + \frac{1}{24} \sum_{i=1}^{n_d} \left(\frac{\partial^4 g}{\partial x_i^4} \right) \Gamma_{x_i} \sigma_{x_i}^4 + \frac{1}{8} \sum_{i=1}^{n_d} \sum_{\substack{j=1 \\ i \neq j}}^{n_d} \left(\frac{\partial^4 g}{\partial x_i^2 \partial x_j^2} \right) \sigma_{x_i}^2 \sigma_{x_j}^2 + O(\boldsymbol{\sigma}_x^5)$$

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Introduction to the URQ approach

- System of eight equations, its solution leads to:
- Optimal values of scaling parameter ***h***

$$h_{1_i} = \frac{\gamma_{X_i}}{2} + \sqrt{\Gamma_{X_i} - \frac{3\gamma_{X_i}}{4}}$$

$$h_{2_i} = \frac{\gamma_{X_i}}{2} - \sqrt{\Gamma_{X_i} - \frac{3\gamma_{X_i}}{4}}$$

$\gamma_{\boldsymbol{X}}$ … skewness of ***X***
$\Gamma_{\boldsymbol{X}}$ … kurtosis of ***X***

- Optimal values of weights ***w***

Mean

$$w_0^{(\mu)} = 1 + \sum_{i=1}^{n_d} \frac{1}{h_{1_i} h_{2_i}}$$

$$w_{1_i}^{(\mu)} = \frac{1}{h_{1_i}\left(h_{1_i} - h_{2_i}\right)}$$

$$w_{2_i}^{(\mu)} = -\frac{1}{h_{2_i}\left(h_{1_i} - h_{2_i}\right)}$$

Variance

$$w_{0_i}^{(\sigma)} = \frac{2}{h_{1_i} h_{2_i}\left(h_{1_i} - h_{2_i}\right)^2}$$

$$w_{1_i}^{(\sigma)} = \frac{\left(h_{1_i}\right)^2 - h_{1_i} h_{2_i} - 1}{\left(h_{1_i}\right)^2\left(h_{1_i} - h_{2_i}\right)^2}$$

$$w_{2_i}^{(\sigma)} = \frac{\left(h_{2_i}\right)^2 - h_{1_i} h_{2_i} - 1}{\left(h_{2_i}\right)^2\left(h_{1_i} - h_{2_i}\right)^2}$$

Introduction to the URQ approach

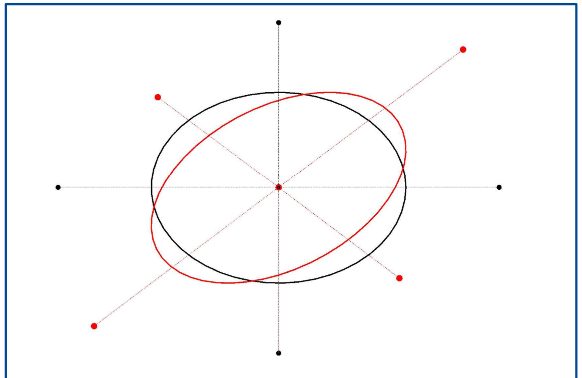
- **Correlation of input parameters**
  - spectral decomposition:

$$\boldsymbol{\Sigma}_{XX} = \boldsymbol{V}\boldsymbol{D}\boldsymbol{V}^{-1}$$

  - $\boldsymbol{\Sigma}_{XX}$ … input covariance matrix
  - $\boldsymbol{V}$ … right eigenvector of $\boldsymbol{\Sigma}_{XX}$
  - $\boldsymbol{D}$ … diagonal matrix of eigenvalues of $\boldsymbol{\Sigma}_{XX}$

$$\boldsymbol{C} = \boldsymbol{V}\sqrt{\boldsymbol{D}}$$

  - URQ nodes with correlation:



uncorrelated

$$\begin{aligned} \boldsymbol{x}_0 &= \boldsymbol{\mu}_X \\ \boldsymbol{x}_{1_i} &= \boldsymbol{\mu}_X + h_{1_i}\,\sigma_{X_i}\,\boldsymbol{e}_i \\ \boldsymbol{x}_{2_i} &= \boldsymbol{\mu}_X + h_{2_i}\,\sigma_{X_i}\,\boldsymbol{e}_i \end{aligned}$$

correlated

$$\begin{aligned} \boldsymbol{x}_0 &= \boldsymbol{\mu}_X \\ \boldsymbol{x}_{1_i} &= \boldsymbol{\mu}_X + h_{1_i}\boldsymbol{C}\,\boldsymbol{e}_i \\ \boldsymbol{x}_{2_i} &= \boldsymbol{\mu}_X + h_{2_i}\boldsymbol{C}\,\boldsymbol{e}_i \end{aligned}$$

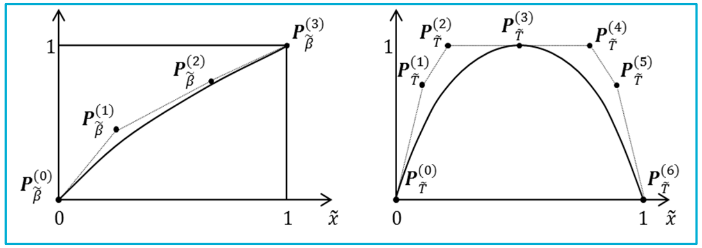
Introduction to the URQ approach

This approach is called the

Univariate	**(no dependencies between parameters - only one dimension is altered to obtain each node → Spectral decomposition)**
Reduced	**(as few nodes as possible, Taylor is truncated)**
Quadrature	**(approximation of an integral via a sum)**

Method

Performance of the URQ



- **Compare URQ and oLHC by using CFD example**
- **Performance:**
  - DoE with 19 different compressor blade profiles
  - each profile is defined by 14 deterministic design parameters $\boxed{\boldsymbol{p} = [p_1 \dots p_{14}]^T \in \mathbb{R}^{14}}$

$$t = \frac{T_{max}}{c}$$

$$p_{12} = \frac{t - t_{min}}{t_{max} - t_{min}}$$

$$p_{13} = \frac{\beta_{LE} - \beta_{LE\,min}}{\beta_{LE\,max} - \beta_{LE\,min}}$$

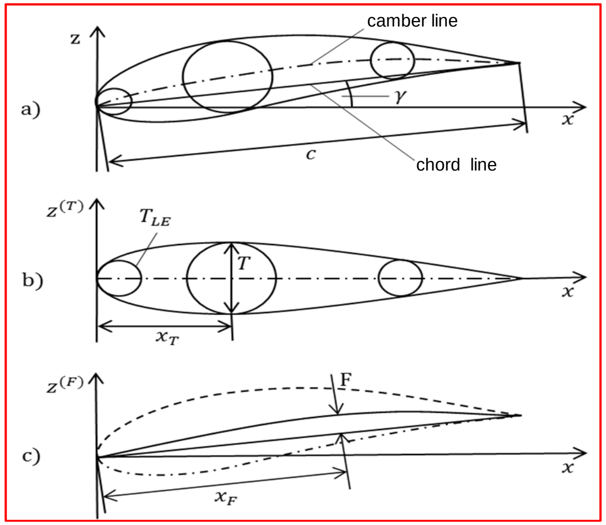
$$p_{14} = \frac{\beta_{TE} - \beta_{TE\,min}}{\beta_{TE\,max} - \beta_{TE\,min}}$$

  - uncertainty defined with 8 parameter → $n_d = 8$

$$\boxed{\boldsymbol{d} = [\gamma \quad c \quad T_{max} \quad x_{T_{max}} \quad F_{max} \quad x_{F_{max}} \quad T_{LE} \quad T_{TE}] \in \mathbb{R}^8}$$

where $\boldsymbol{d} = \boldsymbol{d}(\boldsymbol{p})$

  - determine following system responses:
    - mean of pressure loss
    - variance of pressure loss
    - mean of exit flow angle
    - variance of exit flow angle



Question: „How much additional cost is needed to generate equivalent estimates?"

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Performance of the URQ

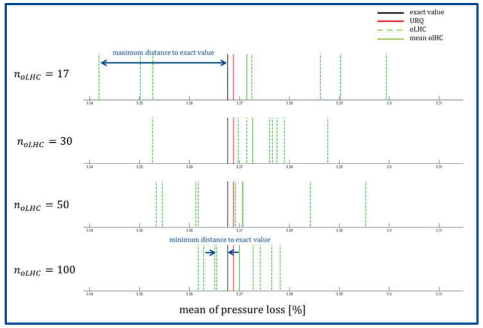
- **Accomplishment:**
  - reference (exact value) is oLHC with $N = 10.000$
  - number of nodes for URQ not varying:

    $$n_{URQ} = 2n_d + 1 = 17$$

  - number of nodes for oLHC varying:

    $$n_{oLHC} = 17 \text{ (equivalent cost)}, 30, 50 \text{ and } 100$$

  - each oLHC- calculation was repeated 10x to show response variation :



→10 solution, out of that

- 1 run with <u>minimal</u> distance to optimal solution
- 1 run with <u>maximal</u> distance to optimal solution

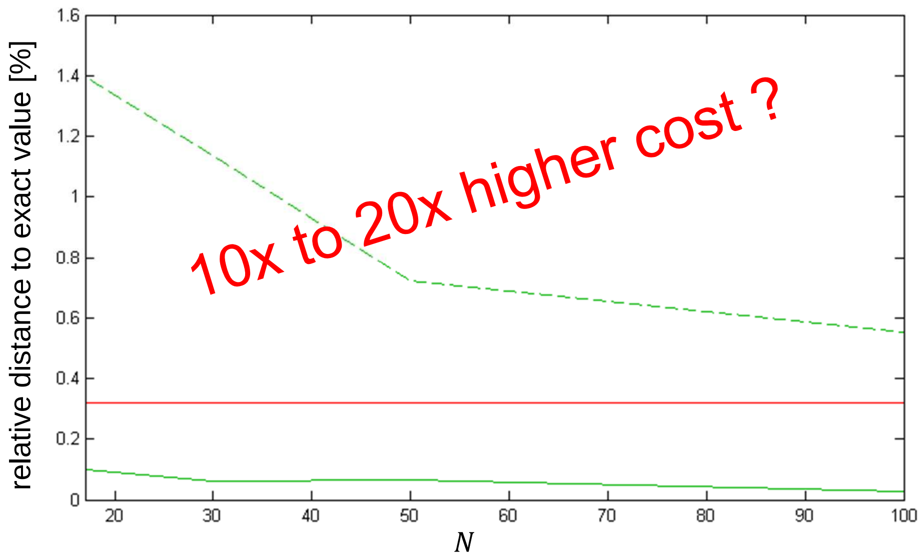
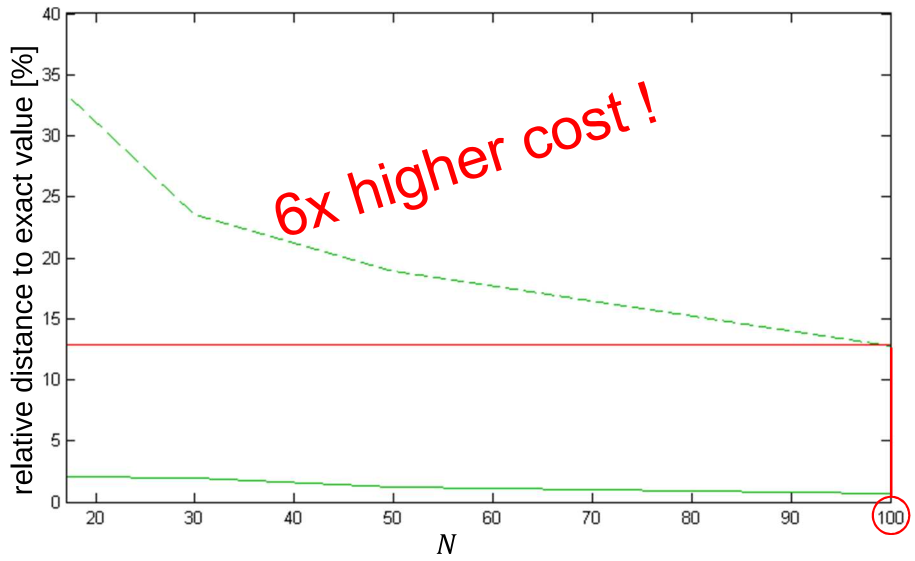
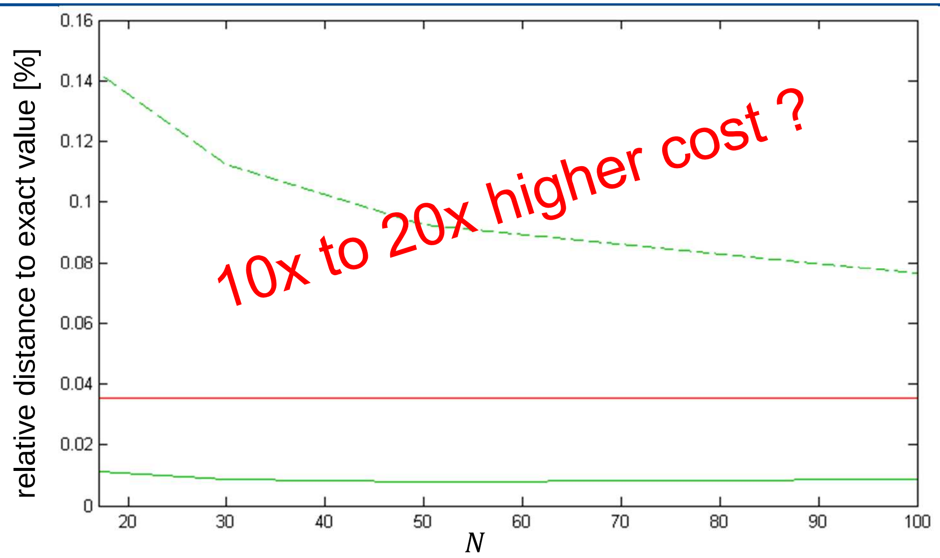
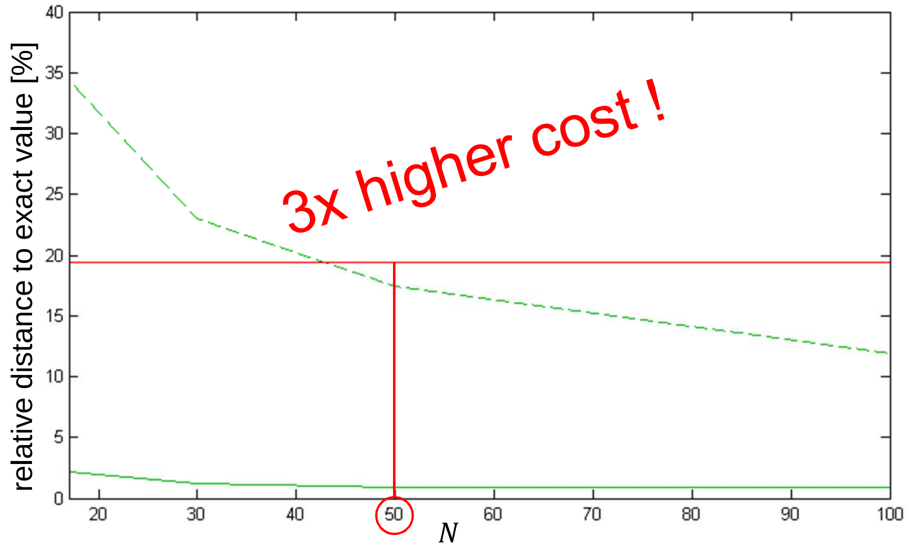
- build average out of all solutions of $n_{DoE} = 19$ profiles

  → 1 average out of 19 profiles with <u>minimal</u> distance

  → 1 average out of 19 profiles with <u>maximal</u> distance

Performance of the URQ

URQ
oLHC (runs with <u>min</u> distance to optimal solution)
oLHC (runs with <u>max</u> distance to optimal solution)

	mean	variance
pressure loss		
exit flow angle		

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Performance of the URQ

- 10- to 20-times higher cost for oLHC- simulation to get equivalent approximations for the mean of system response
- 3- to 6-times higher cost for oLHC- simulation to get equivalent approximations for the variance of system response

? Why is there a difference between the approximation- quality of mean and variance ?

- **Reason:** Comparison of coefficients between Gaussian- Quadrature and Taylor-series expansion doesn‘t feature cross-derivative terms of the Taylor-series.

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Performance of the URQ

➢ **Approximations of system responses with Taylor- series**

- Mean μ_g

$$\mu_g = E[g(\boldsymbol{x})] = \int_{-\infty}^{+\infty} g(\boldsymbol{x})\, f_{\boldsymbol{X}}(\boldsymbol{x})\, d\boldsymbol{x}$$

$$= \overbrace{g(\boldsymbol{\mu_X})}^{M_1} + \overbrace{\frac{1}{2}\sum_{i=1}^{n_d}\left(\frac{\partial^2 g}{\partial x_i^2}\right)\sigma_{x_i}^2}^{M_2} + \overbrace{\frac{1}{6}\sum_{i=1}^{n_d}\left(\frac{\partial^3 g}{\partial x_i^3}\right)\gamma_{x_i}\sigma_{x_i}^3}^{M_3} + \overbrace{\frac{1}{24}\sum_{i=1}^{n_d}\left(\frac{\partial^4 g}{\partial x_i^4}\right)\Gamma_{x_i}\sigma_{x_i}^4}^{M_4} + \overbrace{\frac{1}{8}\sum_{i=1}^{n_d}\sum_{\substack{j=1\\ i\neq j}}^{n_d}\left(\frac{\partial^4 g}{\partial x_i^2 \partial x_j^2}\right)\sigma_{x_i}^2\sigma_{x_j}^2}^{M_5} + O(\boldsymbol{\sigma_x^5})$$

- Variance σ_g^2

$$\sigma_g^2 = E\left[\left(g(\boldsymbol{x}) - \mu_g\right)^2\right] = \int_{-\infty}^{+\infty} \left(g(\boldsymbol{x}) - \mu_g\right)^2 f_{\boldsymbol{X}}(\boldsymbol{x})\, d\boldsymbol{x}$$

$$= \overbrace{\sum_{i=1}^{n_d}\left(\frac{\partial g}{\partial x_i}\right)\sigma_{x_i}^2}^{S_1} + \overbrace{\sum_{i=1}^{n_d}\left(\frac{\partial^2 g}{\partial x_i^2}\right)\left(\frac{\partial g}{\partial x_i}\right)\gamma_{x_i}\sigma_{x_i}^3}^{S_2} + \overbrace{\sum_{i=1}^{n_d}\sum_{\substack{j=1\\ i\neq j}}^{n_d}\left(\frac{\partial^3 g}{\partial x_i^2\, \partial x_j}\right)\left(\frac{\partial g}{\partial x_j}\right)\sigma_{x_i}^2\sigma_{x_j}^2}^{S_3}$$

$$+ \overbrace{\frac{1}{2}\sum_{i=1}^{n_d}\sum_{\substack{j=1\\ i\neq j}}^{n_d}\left(\frac{\partial^2 g}{\partial x_i\, \partial x_j}\right)^2 \sigma_{x_i}^2\sigma_{x_j}^2}^{S_4} + \overbrace{\frac{1}{3}\sum_{i=1}^{n_d}\left(\frac{\partial^3 g}{\partial x_i^3}\right)\left(\frac{\partial g}{\partial x_i}\right)\Gamma_{x_i}\sigma_{x_i}^4}^{S_5} + \overbrace{\frac{1}{4}\sum_{i=1}^{n_d}\left(\frac{\partial^2 g}{\partial x_i^2}\right)^2 \left(\Gamma_{x_i} - 1\right)\sigma_{x_i}^4}^{S_6} + O(\boldsymbol{\sigma_x^5})$$

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Validation of the error

- Exact response mean and variance are calculated analytically for a test function together with a uniform probability distribution
- The input standard deviation is repeatedly halved
- Result: error ratios converge to 16
  $\rightarrow$ Error of order $O(\sigma_x^4)$

$\varepsilon_{\mu_f}(\boldsymbol{\sigma}_x)$ ratio	$\frac{\varepsilon_{\mu_f}(0.32)}{\varepsilon_{\mu_f}(0.16)}$	$\frac{\varepsilon_{\mu_f}(0.16)}{\varepsilon_{\mu_f}(0.08)}$	$\frac{\varepsilon_{\mu_f}(0.08)}{\varepsilon_{\mu_f}(0.04)}$	$\frac{\varepsilon_{\mu_f}(0.04)}{\varepsilon_{\mu_f}(0.02)}$
Value	15.63	15.91	15.98	15.99

$\varepsilon_{\sigma_f^2}(\boldsymbol{\sigma}_x)$ ratio	$\frac{\varepsilon_{\sigma_f^2}(0.32)}{\varepsilon_{\sigma_f^2}(0.16)}$	$\frac{\varepsilon_{\sigma_f^2}(0.16)}{\varepsilon_{\sigma_f^2}(0.08)}$	$\frac{\varepsilon_{\sigma_f^2}(0.08)}{\varepsilon_{\sigma_f^2}(0.04)}$	$\frac{\varepsilon_{\sigma_f^2}(0.04)}{\varepsilon_{\sigma_f^2}(0.02)}$
Value	15.34	15.83	15.96	15.99

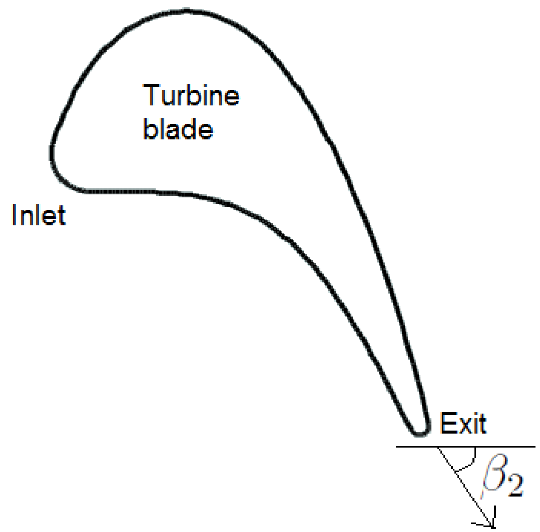
Aerodynamic blade-to-blade robust turbine blade optimization

- **Problem definition:**

$$\min_{d \in F} P(\omega > \omega^{nom})$$

subject to

$$P(|\beta_2 - {\beta_2}^{tar}| > \varepsilon) < p^{lim}$$
$$P\left(d > d^{lim}\right) < p^{lim}$$
$$P\left(H^{TE,SS} > H^{lim}\right) < p^{lim}$$
$$P\left(M_{max} > M^{lim}\right) < p^{lim}$$



$$\omega = \frac{p_{0,E}^{isen} - \bar{p}_{0,E}}{p_{0,I} - p_I}$$

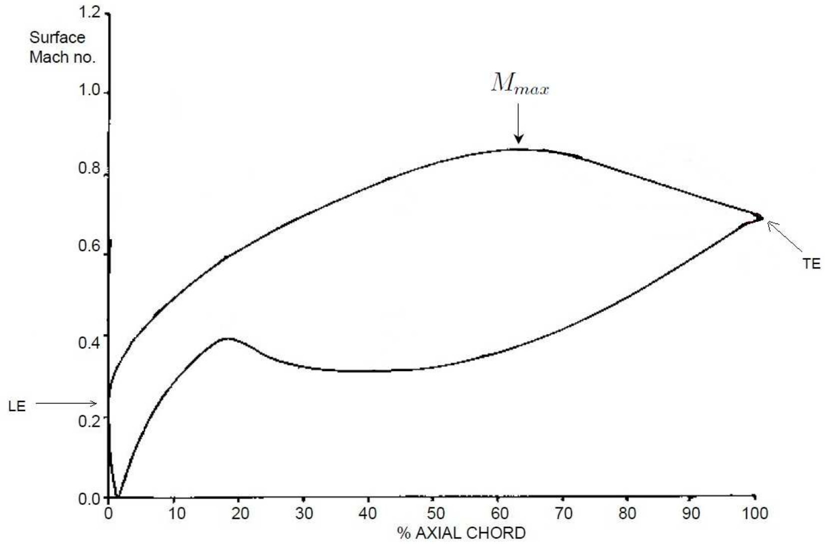
where

$p_{0,E}^{isen}$ … isentropic total pressure at exit plane
$\bar{p}_{0,E}$ … mass-averaged exit stagnation pressure
$p_{0,I}$ … total pressure at inlet plane
p_I … static pressure at inlet plane

Aerodynamic blade-to-blade robust turbine blade optimization

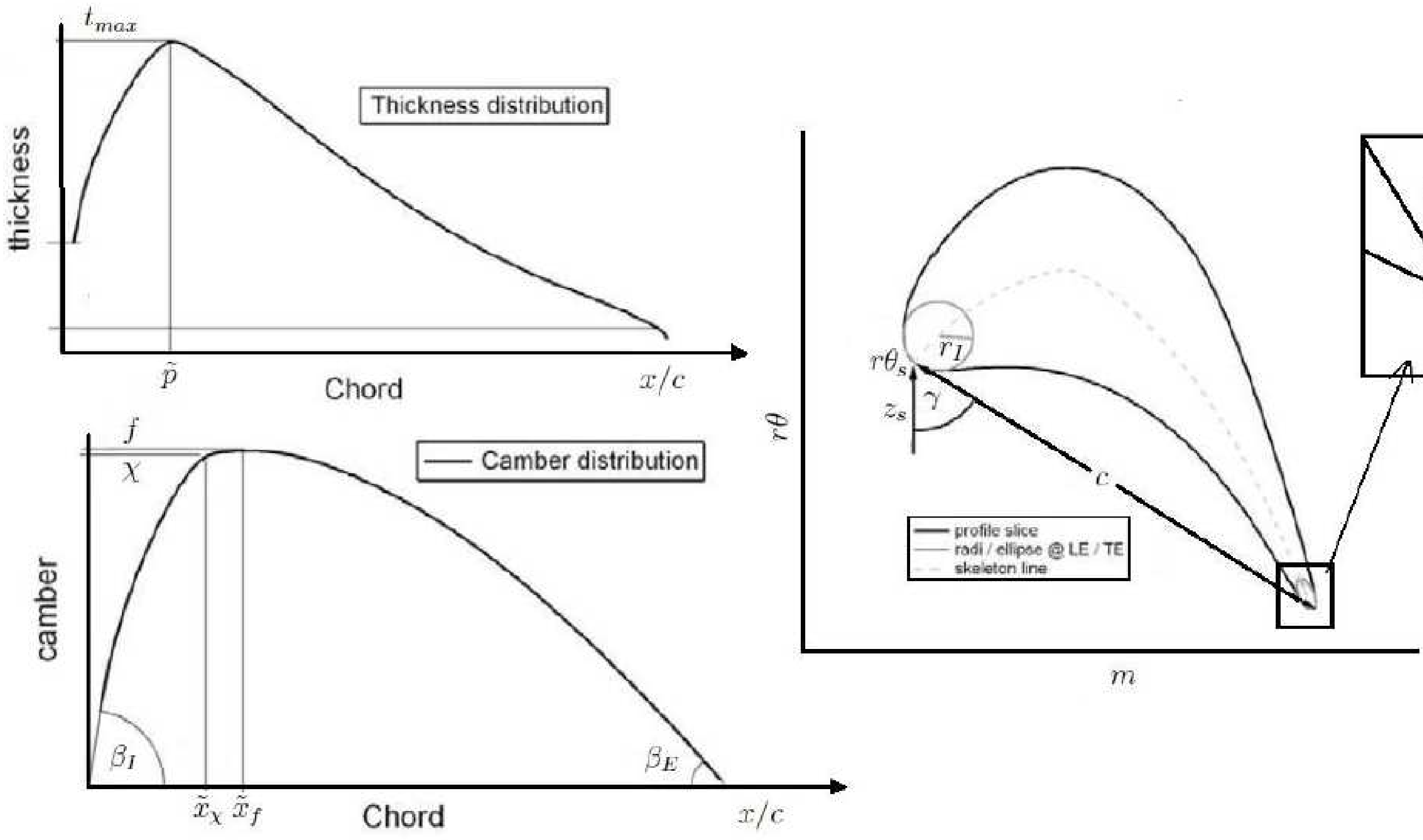
- Back surface diffusion *d* is the diffusion from M_{max} to *TE*.

- High shape factors *H* indicate flow separation.
  $H^{TE,SS} < 2$ to insure there is no separation when flow reaches next vane.



Aerodynamic blade-to-blade robust turbine blade optimization

- „NACA“ parameterization:



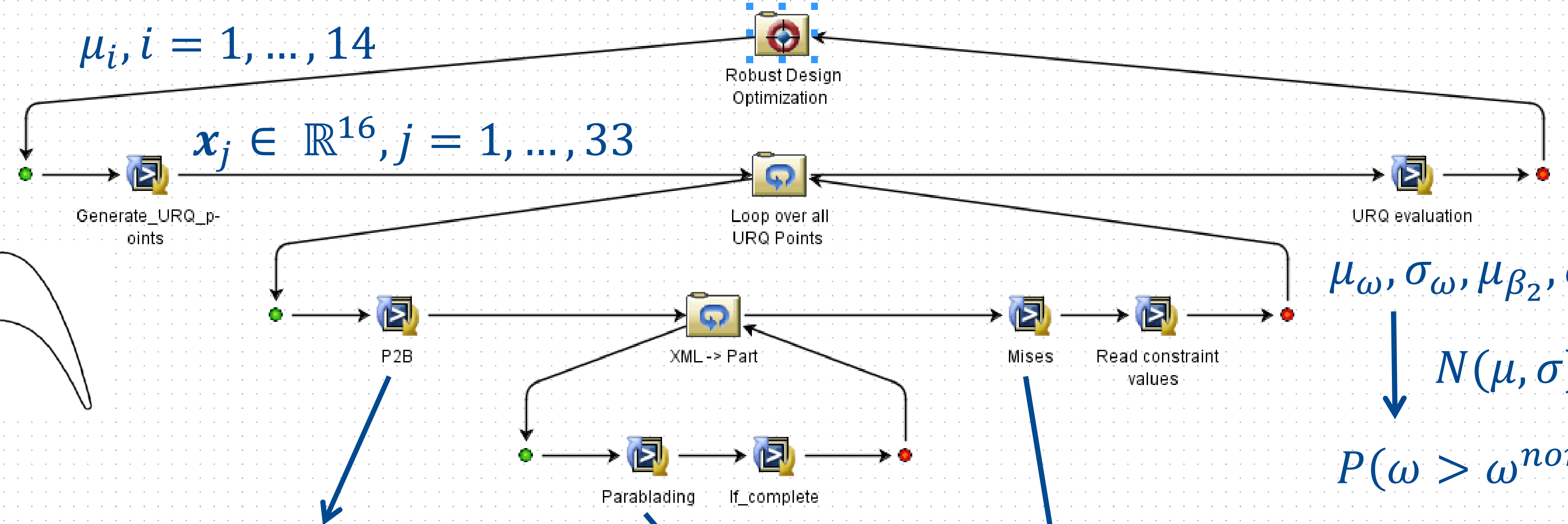
Aerodynamic blade-to-blade robust turbine blade optimization

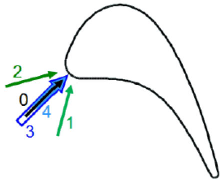
- Isight process:

RR customized CMA-ES algorithm

$\mu_i, i = 1, \ldots, 14$

$\boldsymbol{x}_j \in \mathbb{R}^{16}, j = 1, \ldots, 33$





$\mu_\omega, \sigma_\omega, \mu_{\beta_2}, \sigma_{\beta_2}, \ldots$

$N(\mu, \sigma)$

$P(\omega > \omega^{nom}), \ldots$


```
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            <y unit="mm">0.0161232 -0.33589
            <z unit="mm">249.305 249.312 24
        </leading_edge>
        <trailing_edge>
            <x unit="mm">10.0797 10.0977 10
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            <z unit="mm">246.917 246.914 24
        </trailing_edge>
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```


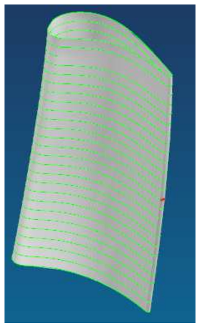

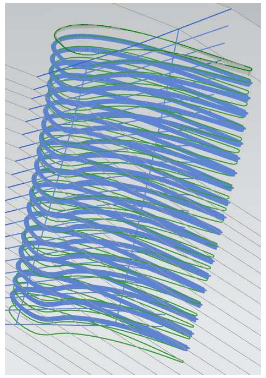

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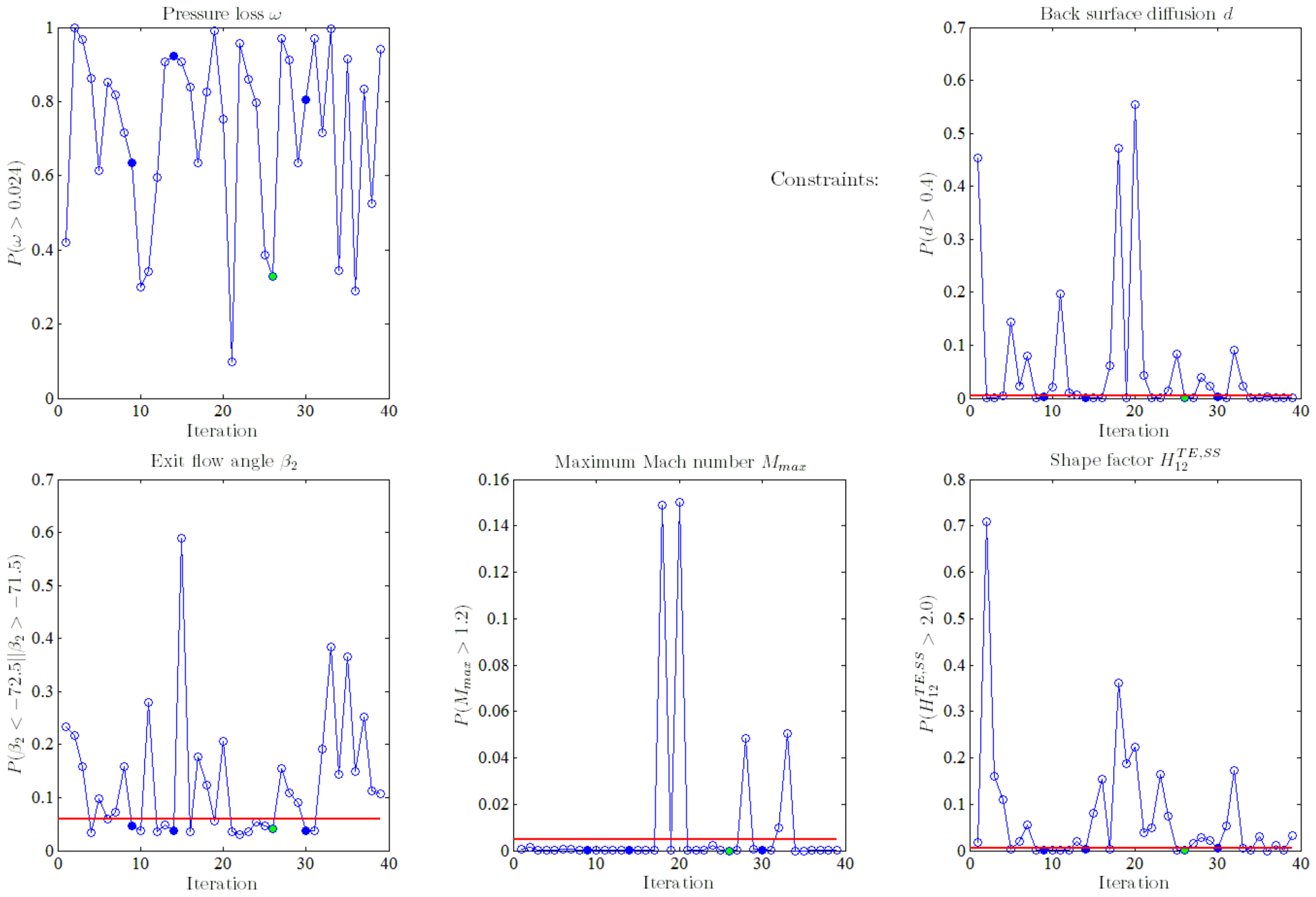
First results of turbine optimization

- First 39 CMA-ES optimization runs
- Filled dots: All constraints satisfied; Green dot: Optimal solution
- Wall clock time per run: approx. 30 min.



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Summary of URQ method

- no tuning parameters
- only depending of the first four statistical moments of design parameters
- skewness and kurtosis of design parameters are covered
- correlation between design parameter can be included
- error of order $O(\sigma_{\boldsymbol{x}}^{4})$
- computationally much cheaper than oLHC

Thank you for your attention!